FLUID MOMENTUM TRANSFER ATTITUDE CONTROL

OF SPACE VEHICLES

by

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Submitted to the Department of Naval Architecture and Marine Engineering in partial fulfillment of the requirements for the Degree of Master of Science in Mechanical Engineering and the Professional Degree, Naval Engineer.

ABSTRACT

The feasibility of using a fluid momentum transfer device to control the attitude of space vehicles is investigated and trends in the variation of system weight associated with the use of fluids with different physical properties are determined.

The results indicate that devices of this type can provide adequate control with good response time and no steady state position error if they are used in conjunction with an auxiliary mass expulsion system. The principal advantage of this type of system is that it can provide the torques necessary to maneuver the vehicle and to cancel disturbance torques with time average values of zero without expelling mass from the vehicle. This reduces the amount of fuel that has to be carried and can greatly extend the length of missions.

Information is presented on the design of fluid momentum transfer attitude control systems as well as information pertaining to the relative merits of the different fluids that can be used.

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CHAPTER 1

INTRODUCTION

1.1 General Introduction to Space Vehicle Attitude Control

As a vehicle orbits the earth or moves from point to point in space its angular position with respect to its moving reference frame must be precisely controlled if it is to perform any useful function. Just how precise this control must be is dependent on the type of mission that the vehicle is to perform. In general it is required that the vehicle be able to rotate about three mutually perpendicular axes. These axes and the nomenclature associated with them are shown in Figure (1.1).

1.2 Purpose and Scope of Thesis

The object of this investigation is to study the feasibility of controlling a space vehicle's attitude by using a fluid momentum transfer device. Until now attitude control of large space vehicles generally has been accomplished by means of mass expulsion systems. Mass expulsion systems provide the necessary control torques by expelling mass through nozzles exterior to the vehicle. This is an effective means of control except that it requires that the vehicle carry the mass to be expelled along with it. For extremely long missions a great deal of mass would have to be carried. Because of this reason, it is desirable to have a system which can control the attitude effectively for long missions and at the same time does not require the consumption of excessive amounts of fuel. Momentum storage devices appear to be a solution to this problem, since they can provide control without expelling large amounts of mass from the vehicle.

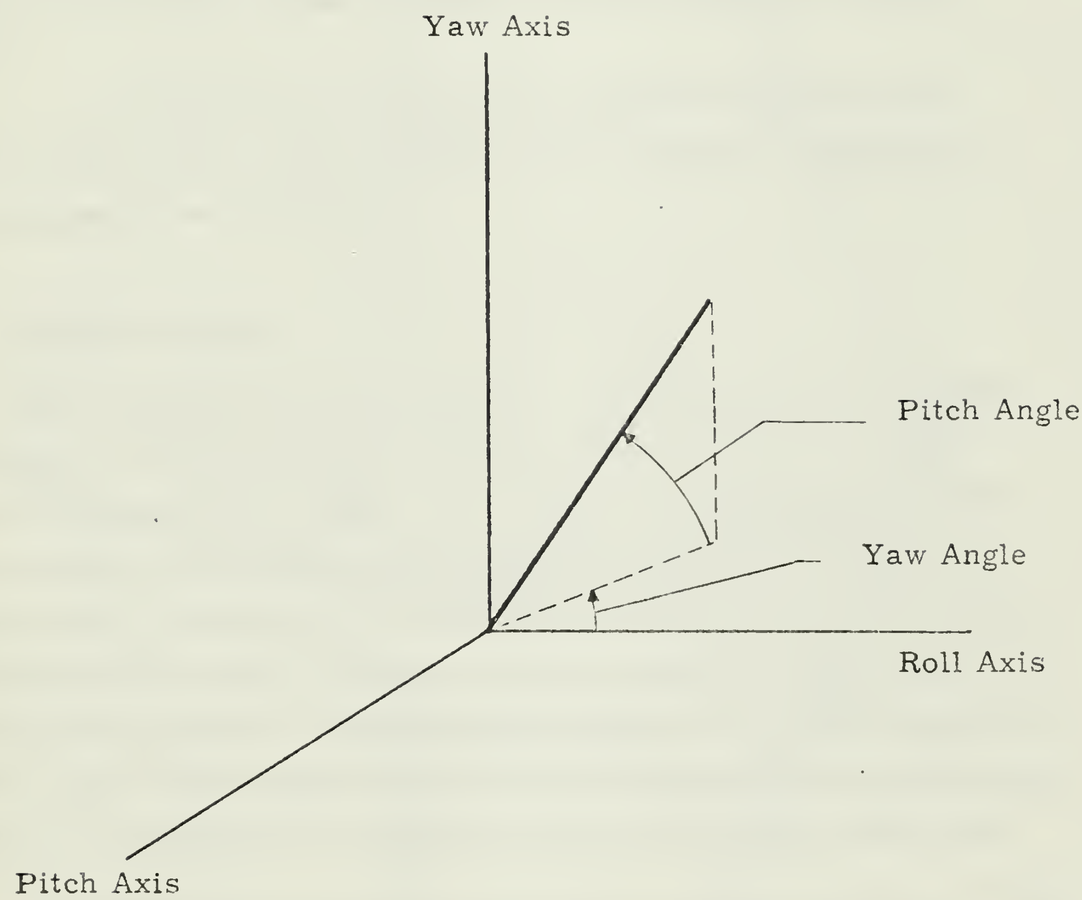

FIGURE (1. 1) COORDINATE SYSTEM

The specific objectives of this investigation are:

1.) To study the dynamics of a fluid momentum transfer attitude control system.

2.) To develop a means of providing control which will enable the vehicle to maneuver and to maintain attitude in the presence of outside disturbances.

3.) To present information about how the use of different fluids effects the system weight of a fluid momentum transfer device.

4.) To determine any trends associated with how the system weight varies with the design of the momentum transfer device and control requirements.

1.3 Proposed System

The proposed system consists of three separate closed loops of tubing which run around the perimeter of the vehicle. The loops are so arranged that the planes of the loops are perpendicular to the three principal axes of the vehicle and are therefore perpendicular to each other. See Figure (1.2). Each loop is filled with a fluid which can be pumped in either direction around the loop by a separately controlled pumping device. Accelerating the fluid in one of the loops produces a torque on the vehicle about the axis perpendicular to the plane of that loop and opposite in direction to the acceleration of the fluid.

The system is used to produce torques for two purposes. First, the torques produced can cause the vehicle to be rotated to a new position, thus maneuvering the vehicle. Second, the torques produced can be used to cancel out the rotations caused by externally or internally applied torques, thus maintaining the attitude of the vehicle.

Disturbance torques can be broken down into two basic classes, those which are periodic and those which occur randomly. Some of the periodic torques will act in one direction as much as they will in the

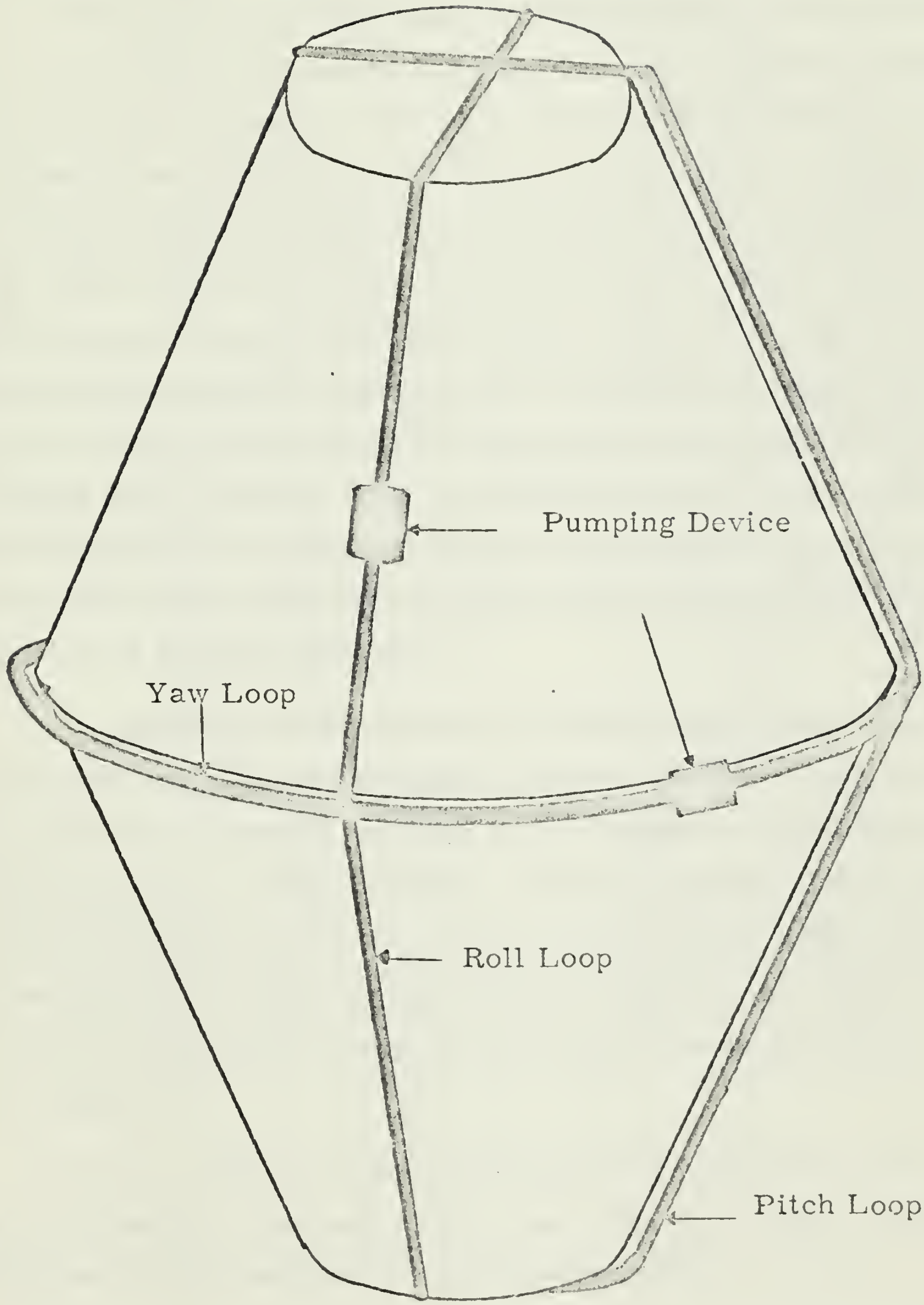

FIGURE (1. 2) PROPOSED LOOP ARRANGEMENT

opposite, and therefore the time average torque over the period will be zero. These torques will require that the fluid in the loops be accelerated as much in one direction as it is in the other to maintain vehicle position. Thus there is no net change in fluid velocity from period to period. However the remaining disturbance torques act in one direction more than in the other, and therefore will require the fluid to be accelerated in one direction more than it is in the other, resulting in an increase in fluid velocity. Over a period of time the fluid velocity in the loops will build up to a considerable level. When the fluid velocity is high enough so that the pressure drop in the loop is equal to the maximum pressure that the pump is capable of supplying the fluid can no longer be accelerated and the fluid has reached its maximum steady state velocity. This is called saturation. When saturation occurs the system is no longer capable of providing control. In order to prevent this from occuring the system must be equipped with an auxiliary mass expulsion system.

This auxiliary system allows the fluid velocity to be reset to zero by producing a torque opposite to the one needed to decelerate the fluid. After the system has been reset it again is capable of supplying the necessary attitude control. With the type of system proposed it would be necessary for the vehicle to carry only enough fuel for the mass expulsion system to cancel out the disturbance torques which have a net average value. Disturbance torques are discussed more fully in Chapter 2.

The loops of the circulating fluid act very much like a fly-wheel and throughout the rest of this report the loop, fluid, and pump combination will be referred to as a fluid flywheel.

The basic control system block diagram is shown in Figure (1.3). This shows the relationship between the torques produced by the control system and the disturbance torques as well as the inter-relationship of the various components in the proposed system.

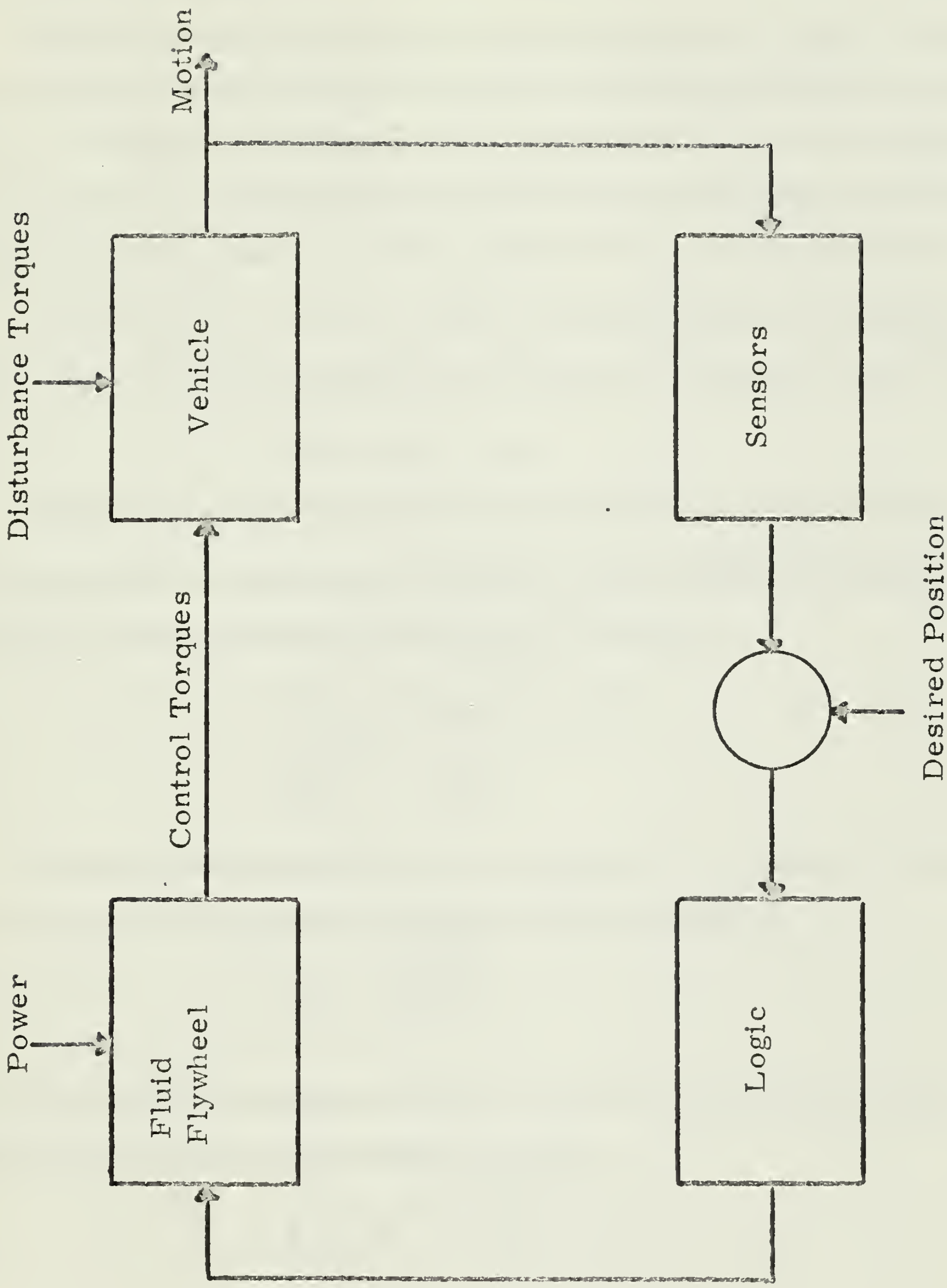

FIGURE (1.3) BASIC CONTROL SYSTEM BLOCK DIAGRAM

1.4 Theory of Operation

The law of conservation of angular momentum states "when a body or system of bodies is not acted upon by an unbalanced external torque, the angular momentum of the system about any fixed axis remains constant". Considering one axis of the vehicle and its associated fluid flywheel, this law can be applied in the following manner.

Assuming that both the vehicle and the flywheel are initially at rest, then the angular momentum about any particular axis is zero.

$$H = I\,\omega = 0$$

(Symbols with associated units are defined in Appendix F)

The total angular momentum is the sum of the angular momentum of the vehicle and the angular momentum of the fluid.

$$H_{total} = I_f\omega_f + I_v\omega_v = 0$$

$$I_f\omega_f = -I_v\omega_v$$

If the fluid is accelerated to a velocity of ω_f then the vehicle must rotate in the opposite direction at the velocity ω_v.

$$\omega_v = \frac{-I_f\omega_f}{I_v}$$

Now, if the fluid is stopped at time t, the vehicle must also stop. Integrating both sides with respect to time:

$$\int_0^t \omega_v\,dt = \frac{-I_f}{I_v}\int_0^t \omega_f\,dt$$

where $\int_0^t \omega_v\,dt = \Phi_v$, and $\int_0^t \omega_f\,dt = \Phi_f$.

Therefore, it can be seen that by rotating the fluid an angle of Φ_f that the vehicle rotates to an angle of Φ_v in the opposite direction.

Assume now that an external torque is applied to the system. The vehicle will rotate and since the fluid is frictionally coupled to the

vehicle it also will rotate. It is usually desirable to eliminate the rotation of the vehicle caused by this external torque. Now the total angular momentum of the system is no longer equal to zero, but is equal to some vector quantity, H. It can be seen by the same reasoning as above that if the fluid were accelerated in the direction of the rotating vehicle it would eventually store all the angular momentum, H, in its rotation and the vehicle would stop rotating. However, now the fluid must be kept moving at a constant velocity. If the fluid were to stop the momentum stored in the flywheel would be transferred back to the vehicle and the vehicle would start rotating again.

As mentioned in Section (1. 3) there is a fluid velocity at which saturation occurs, therefore the system can only absorb a finite amount of angular impulse. The amount of angular impulse that the system can absorb is equal to the amount of angular momentum stored in the flywheel at saturation. This implies that a small torque acting for a long period of time or a large torque acting for a relatively short period of time will cause saturation.

For the purpose of illustration consider the example of a boy standing at the edge of a revolving merry-go-round. If the boy runs around the circumference of the merry-go-round and in the same direction as the rotation, he increases his angular momentum and because the total angular momentum must remain constant, decreases that of the merry-go-round. At some velocity of the boy all of the angular momentum will be transferred to him and the merry-go-round will stop.

The problem is somewhat more complicated here because the space vehicle is free to rotate about three axes, and because of inter-axis coupling effects. Due to inter-axis coupling, if there is fluid motion in the yaw loop and a torque is applied about the pitch axis, the vehicle will not only rotate about the pitch axis but will also rotate about the roll axis. This means that to cancel any disturbance torque, if there is momentum stored in one loop, all three of the fluid flywheels will

probably have to be used.

It is important to note that the principle of operation is not dependent on the shape of the loop to be used, and in fact, the system will work with any loop configuration. Obviously a circular loop uses the mass of the fluid most effectively, however it may be desirable to use another configuration.

1.5 Reasons For Using Fluids

It is apparent from Section (1.4) that mechanical flywheels can also be used as momentum storage devices. A discussion of their use for an attitude control system follows in Chapter 6. There are specific advantages associated with the use of fluid flywheels, however. Fluidic and flueric systems have many inherent properties that make them especially suited for this application.

a.) Simplicity of the system -

The actual control system consists of tubing, pumping devices, rate and position sensors, and associated electronic or flueric control circuits. All of these components can be simple in design and construction. This simplicity results in reduced cost and increased reliability.

b.) Fewer moving mechanical parts -

The system could actually be designed with no moving parts if an electromechanical pump were used. Explicit information about this type of pump is found in Chapter 4. If a more conventional pump were used it would be the only moving part. This reduces wear and increases system life and reliability which are essential requirements for missions of long duration.

c.) Dual purpose of tubing -

The tubing can be incorporated into the structure of the vehicle and thereby provide the dual function of structural member and

an attitude control system.

d.) Center of vehicle is free -

Since there are no spokes or shafts which might be required by a mechanical flywheel the center of the vehicle would not be obstructed.

e.) Heat transfer medium -

Fluids are suited for high temperature or radiation environments. They could be used as a heat transfer medium for some other system in the vehicle.

CHAPTER 2

CONTROL REQUIREMENTS

2.1 Types of Vehicles

The wide range of requirements for space vehicles and satellites can be grouped into four classes as proposed by Reference (1).

Class A: These are generally small, unmanned vehicles and require little or no control. Vehicles of this class such as the Tiros satellite have been effectively controlled with spin-stabilization.

Class B: This class consists of 400 to 2000 pound vehicles which require control about all three principal axes. The Nimbus meteorological satellite fits into this class.

Class C: This class consists of medium size (2000 to 10000 pounds) vehicles. Again these require control about all three axes.

Class D: This class is reserved for future vehicles and space stations larger than 10000 pounds. As with Classes B and C control must be supplied for all three axes.

Throughout this investigation vehicles of Classes B, C, and D were considered because of their greater control requirements. Before the actual size of the fluid flywheel can be determined, the torques acting on the vehicles and the maneuvering requirements must be investigated.

2.2 Disturbance Torques

There are several types of disturbance torques which act on a space vehicle. It is not the intent of this investigation to determine the magnitude or frequency at which they occur. They are only briefly described here to give the reader some insight into the problem.

Externally Caused Disturbance Torques

a.) Gravity gradient -

A vehicle tends to align its axes of principal moments of inertia with the earth's gravitational field. When the longitudinal axis is inclined to the orbital plane a torque is produced. The size of this torque is dependent upon the vehicle's distance from the center of the earth, the angle of inclination, and the difference between the magnitudes of the principal moments of inertia.

b.) Solar radiation pressure -

This torque is produced if the force vector caused by solar radiation pressure does not pass through the center of mass of the vehicle. The size of this torque is dependent upon the surface area, shape, and mass distribution of the vehicle.

c.) Aerodynamic pressure -

This torque is similar to (b), however the force is produced by aerodynamic pressure. Again the size of the torque depends upon the surface area, shape, and mass distribution of the vehicle as well as the orbital altitude since the aerodynamic pressure decreases with increasing altitude.

d.) Meteorite impact -

When a meteorite strikes a vehicle it imparts a momentum transfer to the vehicle. The size of this depends upon the size of the meteorite and relative velocity of the meteorite and the vehicle.

Internally Caused Disturbance Torques

a.) Interaction of electrical currents in the vehicle with the earth's magnetic field -

A current flowing in any conductor in the vehicle produces an electrical field. This field interacts with the earth's magnetic field and produces a torque on the vehicle. A similar torque is produced by any magnetic field in the vehicle.

b.) Expulsion of matter -

As any matter is ejected from the vehicle (including the radiation of electromagnetic energy) a torque is produced.

c.) Movement of equipment or personnel -

The origin of these torques is self-explanatory and they will occur when an object is accelerated to change its position within the vehicle and again when it is decelerated. The size of the torque is dependent upon the mass of the object being moved, its acceleration and deceleration.

d.) Inter-axis coupling -

These torques are produced by gyroscopic action when the axis of any spinning part attached to the vehicle is rotated.

Some of the disturbance torques mentioned above are cyclic in nature and will produce no net impulse acting on the vehicle over the period. For example, if a man in the vehicle moves his arm, as he accelerates his arm he causes a torque to act on the vehicle. However, he will eventually have to stop moving his arm. When his arm is decelerated it causes a torque in the opposite direction. Regardless of how fast he accelerates or decelerates his arm the net angular impulse imparted to the vehicle is zero. During his arm swing the angular momentum stored in the fluid flywheel must be changed to maintain the vehicle's attitude.

but when his arm has stopped the fluid velocity in the loop returns to the same value as it was before he started to move his arm.

2.3 Maneuvering Requirements

The maneuvering requirements of a space vehicle are normally expressed by the time that it takes to complete a rotation of a specified angle. Obviously these requirements vary greatly with the mission. A manned vehicle that is going to rendezvous and dock must be able to change its position faster than a weather satellite. If the size of a vehicle is known as well as the necessary acceleration for a given maneuver, the amount of momentum to be transferred to the fluid flywheel can be determined.

2.4 Momentum Storage Requirements

The flywheel is to be used to cancel rotations caused by disturbance torques and to supply the necessary torques to maneuver the vehicle. As mentioned before some of the disturbance torques are cyclic in nature and do not cause the momentum storage level of the flywheel to change from period to period. However within any given period there will be a maximum amount of momentum stored in the flywheel caused by these torques. In addition to this, it is desirable to be able to maneuver the vehicle during this period without having to use the auxiliary mass expulsion system to reset the momentum storage level. Therefore the total amount of momentum to be stored in the flywheel should be greater than the sum of the maximum momentum requirements due to the cyclic disturbance torques and the momentum requirements for the most demanding maneuver. With this momentum capacity the reset system would only have to be used to dump the net stored momentum resulting from cancellation of the disturbance torques which have a time average value other than zero for this period.

Reasonable approximations to the maximum values of momentum

storage for different sized vehicles have been determined[1] by summing the momentum requirements for the cancellation of the cyclic disturbance torques and that required by the various vehicle maneuvers.

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[1] K.C. Nichol, Research and Investigation on Satellite Attitude Control, General Electric Company, Technical Report No. AFFDL-TR-64-168 Part II (June 1965), Table 4, page 74.

CHAPTER 3

SYSTEM WEIGHT

3.1 General

A space vehicle is similar to a truck, a plane, or a ship in that the smaller the weight and volume of the propulsion and control machinery the more weight and volume that is left for the pay load. It is therefore of great importance to reduce the size and weight of the attitude control system, while providing adequate control.

3.2 System Weight Optimization

The size and weight of the fluid flywheel attitude control system are greatly dependent on the size, shape, and mass distribution of the vehicle. However once the vehicle has been designed and its principal moments of inertia calculated, the maximum momentum storage requirements can be determined for each flywheel in the manner discussed in Section 2.4. Any fluid flywheel system that can provide the value of H_{max} previously determined becomes a possible candidate for the minimum weight system. An optimization study can be conducted to select the least weight system by using H_{max} as the only constraint and varing all of the other parameters. Section 3.3 discusses the weight of the components of the flywheel and how their weight varies as a function of the geometry of the flywheel, fluid and materials used in the flywheel, and the maximum momentum storage requirement.

3.3 Component Weights

The weight of the system can be broken down into the weights of

the several components of the system. The components considered here are: weight of fluid, weight of tubing, weight of pumping device, weight of pumping device motor, and weight of power supply. For the sake of simplicity the loop and tube configurations were assumed to be circular. Similar relationships could be derived for non-circular shapes.

WEIGHT OF FLUID

$$\text{Weight of fluid} = \frac{\gamma_f \, \pi^2 \, D_1 \, D_2^2}{4} \text{ lbs.} \qquad \text{(B 1)}$$

γ_f in lbs/ft^3 (weight density of fluid)

D_1 , D_2 in ft.

See Figure (3.1).

For any particular value of D_1 the weight of the fluid is uniquely determined by the tube diameter and fluid density.

WEIGHT OF TUBING

$$\text{Weight of tubing} = \pi^2 \gamma_m \, D_1 \, t \, D_2 \text{ lbs.} \qquad \text{(B 2)}$$

γ_m in lbs/ft^3 (weight density for material in tubing)

D_1 , D_2 , t in ft.

As shown in Appendix C the tubing thickness can be kept constant as the diameter varies. There is some question here as to whether the tubing weight should be considered as part of the control system or whether it should be part of the structural weight since it can perform the dual function. It was felt that it may not always be desirable to have the tubing serve as a structural member and therefore it was considered as part of the control system weight.

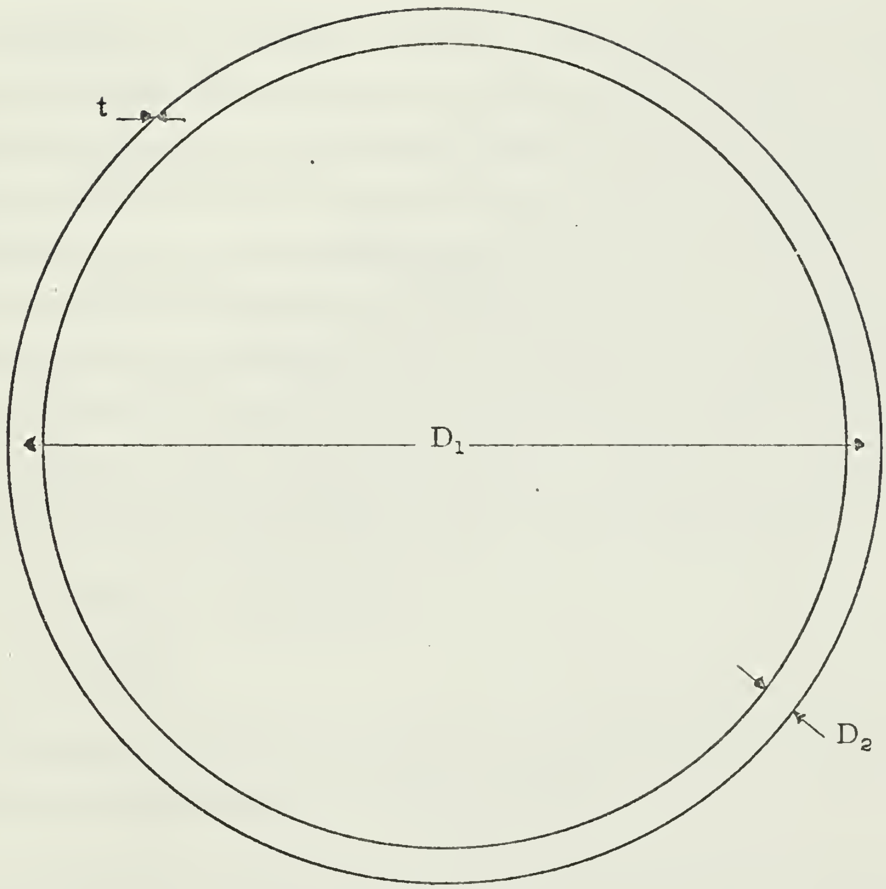

FIGURE (3. 1) FLYWHEEL CONFIGURATION

WEIGHT OF PUMPING DEVICE

In order to determine the weight of the pumping device and how it varies with the system parameters a particular type of pump must be chosen. As discussed in Chapter 4, a survey of pumping devices indicated that the present day pumps are not particularly suited for this application. Because of this no particular type of pump was selected for the purpose of the optimization study.

In general, the weights of pumps vary considerably with the type of construction and the methods of operation. For example, axial flow and centrifugal pumps are generally lighter than screw pumps for the same flow rate. The type of pump that may prove to be the best for this application may vary considerably in construction from the more conventional types and may be considerably heavier. The weight may also vary for the type of fluid chosen. Because of these uncertainties and the fact that no particular type of pump was selected, it was decided to assume a pump weight that would be about the same as a cubic block of steel with its sides equal to the tube diameter.

$$\text{Weight} = 500\ D_2^3 \text{ lbs.}$$

D_2 in ft.

It is felt that this weight would always be larger than the weight of any fluidic or mechanical pump actually selected.

The results of the computer study indicated that although the pump weight affected the total system weight, it had little effect on determining the optimum tube diameter. Once more is known about the design of pumps to be used in the fluid flywheel a more realistic assumption can be made and the weight of the system more closely estimated.

WEIGHT OF PUMP MOTOR

Whatever the type of pump that is selected it must be provided with some type of motor. In electromechanical pumps the magnets can be considered as the motor. For the purposes of this study it was assumed that the pump motor would be a D. C. type. The weight was calculated by fitting a curve to actual values of motor weight versus motor horsepower given in Reference (1) and assuming an overall pump efficiency of 50%.

$$\text{Weight of pump motor} = 0.468\,(\text{Power})^{0.7}_{MAX}\ \text{lbs.}$$

Power in ft-lbs/sec.

WEIGHT OF POWER SUPPLY

The power requirements are extremely difficult to determine. If it were possible to determine every maneuver and every disturbance torque and the time at which they would occur in a mission then the total amount of energy to be supplied could be calculated. From this the actual size of the power supply could be determined. At present not enough information is known to be able to do this accurately. An alternative method can be used[2] to approximate the size of the power supply in which it is assumed that the flywheel would only be operating at its maximum momentum storage mode for 5 percent of the time. During the other 95 percent, the peak required would be about 30 percent of H_{MAX}, giving an average of 15 percent of H_{MAX}. Using these assumptions the average power would be 19.2 percent of the power required for the maximum momentum storage. The weight is calculated assuming 0.225 lbs/watt as a specific weight for the power supply based on a compromise between state of the art solar energy sources and larger more sophisticated power supplies.[3]

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[2] Ibid., page 75; [3] Ibid., page 72.

$$\text{Weight of power supply} = \frac{0.045\,\nu^{.16}\,H_{MAX}^{2.84}}{\rho_f^{1.84}\,D_1^{4.68}\,D_2^{4.84}} \text{ lbs.} \quad (B\ 13)$$

ν in ft^2/sec.

H_{MAX} in ft-lbs/sec.

ρ_f in lbs-sec^2/ft^4

D_1 , D_2 in ft.

The D.C. motor was assumed to have an efficiency of 95% which, when combined with the pump efficiency of 50%, results in an overall efficiency of 47.5%.

3.4 The Optimization Study

A computer program was written to determine the minimum single axis system weight for a Class D vehicle, approximately 10 feet in diameter and with the largest principal moment of inertia of about 10000 slug-feet^2. The study was conducted to determine how the minimum system weight was affected by loop diameter, maximum momentum storage level, and different fluids. The computer calculated the weight for each component using the assumptions of the preceding section and summed them to give the total weight as the tube diameter was allowed to vary. For each of the four fluids used, the minimum system weight was found for three momentum storage levels, and three different loop diameters. The fluids used were mercury, liquid bismuth, NaK (23% sodium, 77% potassium) , and water. These fluids were used because they provide an extremely large range of density and viscosity. For example, the density of mercury is almost 14 times as large as the density of NaK.

The two largest factors in determining the minimum value are the fluid weight and the power supply weight. With small tube diameters

the fluid weight is small and therefore the fluid's moment of inertia is small. In order to achieve the momentum storage capacity the fluid's velocity must be extremely high. The power losses in the fluid are proportional to velocity to the 2.84 power and therefore are extremely large, so the weight of the power supply is large. For large values of tube diameter the fluid weight is very large, however the fluid velocity at the maximum momentum storage mode is small. Therefore by the same reasoning as used above, the losses will be small and the power supply weight is also small.

Figure (3.2) shows how the system weight varies with D_2 for one set of input parameters. The effect of the weights of the power supply and the fluid mentioned in the preceeding paragraph results in a well defined minimum.

The minimum weight and optimum tube diameter for the different momentum storage levels were plotted as functions of loop diameter and are shown in Figures (3.3 to 3.10). The results indicate that both system weight and optimum diameter decrease with increasing loop diameter for all the fluids used. This means that for any given momentum storage requirement that the larger loop diameter results in a smaller system weight. In general the loop diameter will be fixed by the geometry of the vehicle. Increasing the loop diameter beyond that of the vehicle may cause added weight due to the fact that brackets for fixing the tubing to the vehicle would have to be provided. Also since the tubing is attached to the vehicle it is part of the moment of inertia of the vehicle, the larger its diameter the more it increases the vehicle's moment of inertia. This increase in the vehicle's moment of inertia would require a larger momentum storage capability. In general it appears that the loop diameter should be chosen as close as possible to the outside diameter of the vehicle.

The results also showed that for any given momentum storage

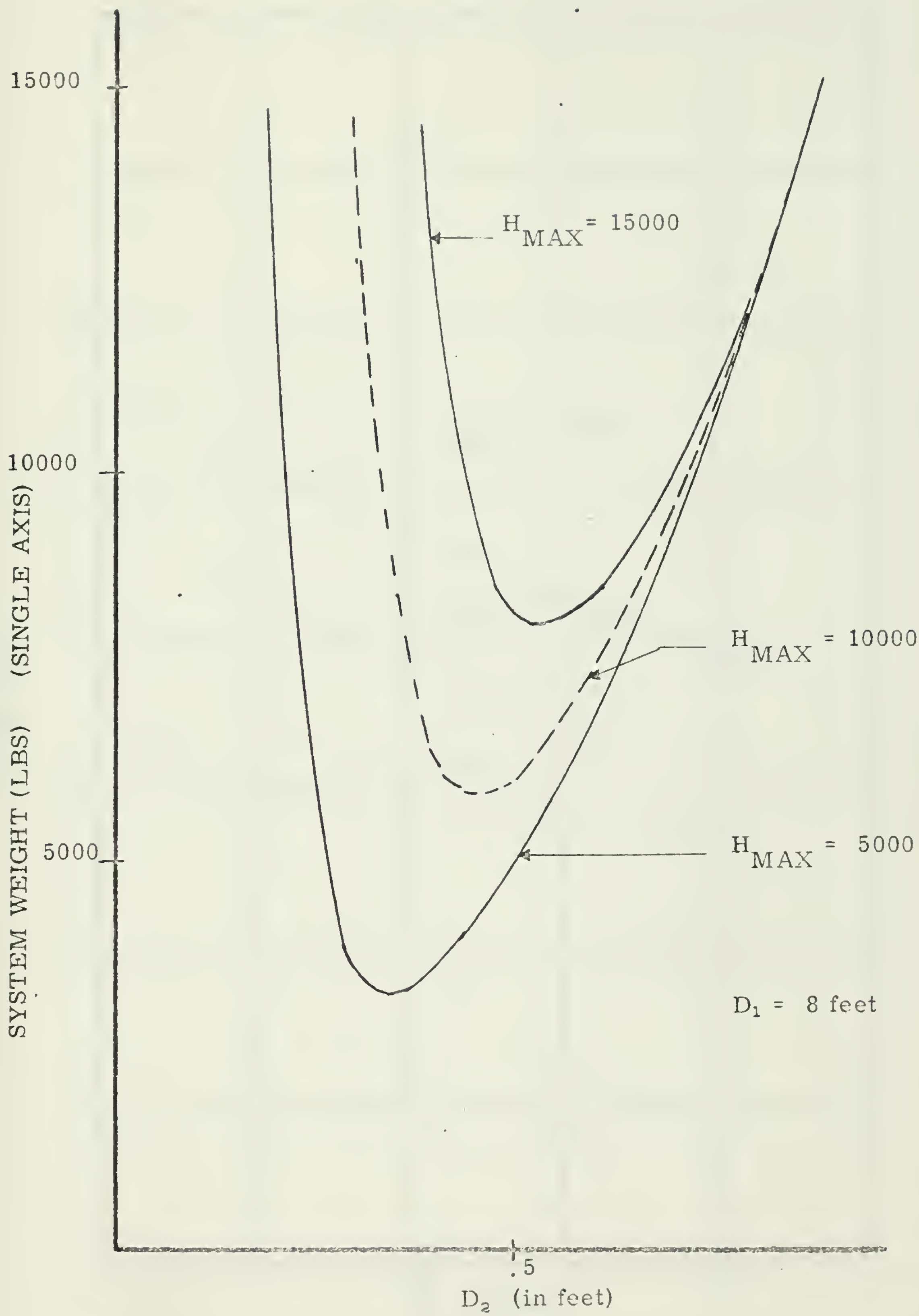
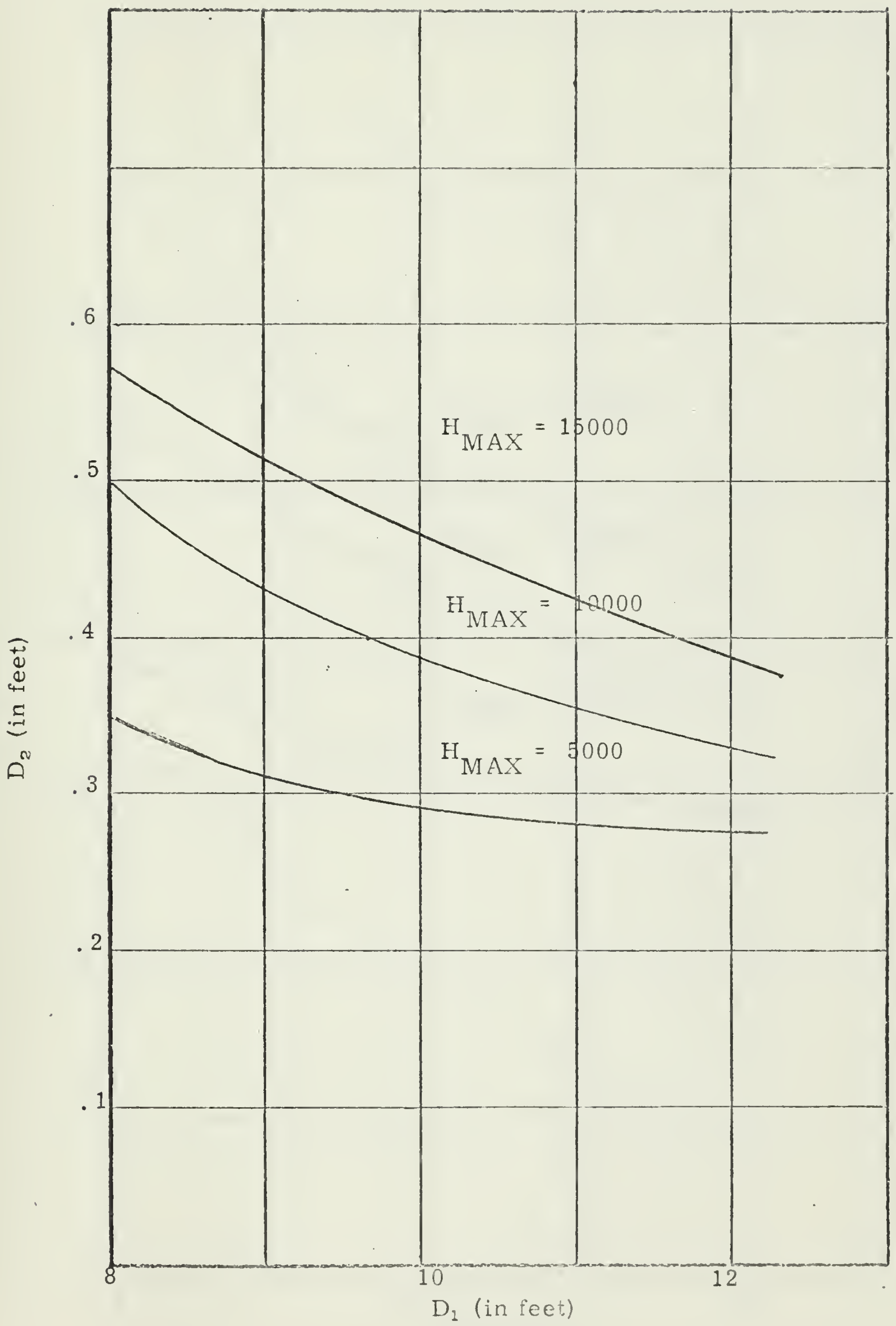
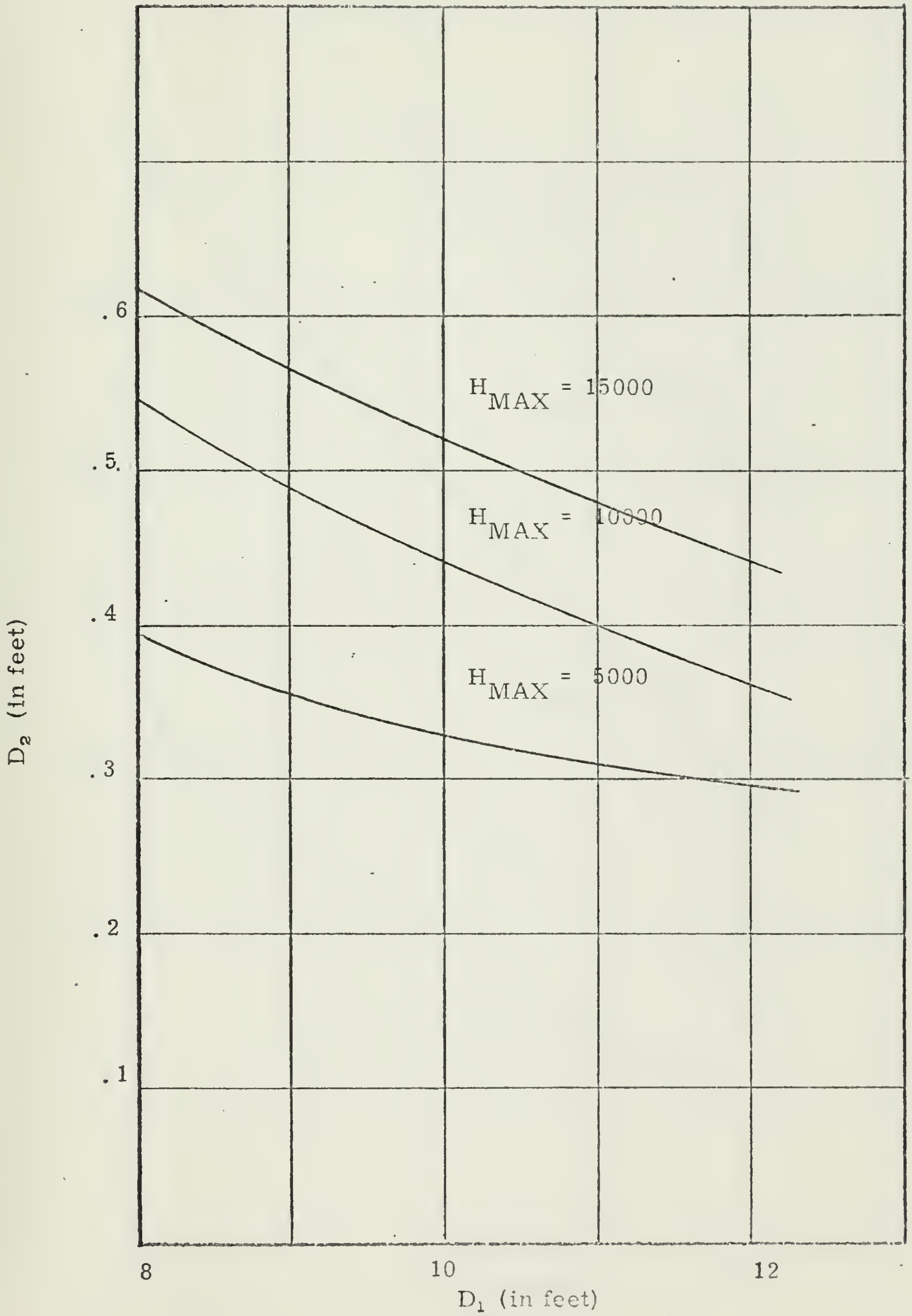


FIGURE (3. 2) SYSTEM WEIGHT AS A FUNCTION OF TUBE DIAMETER



MERCURY

FIGURE (3.3) OPTIMUM TUBE DIAMETER



BISMUTH

FIGURE (3.4) CPTIMUM TUBE DIAMETER

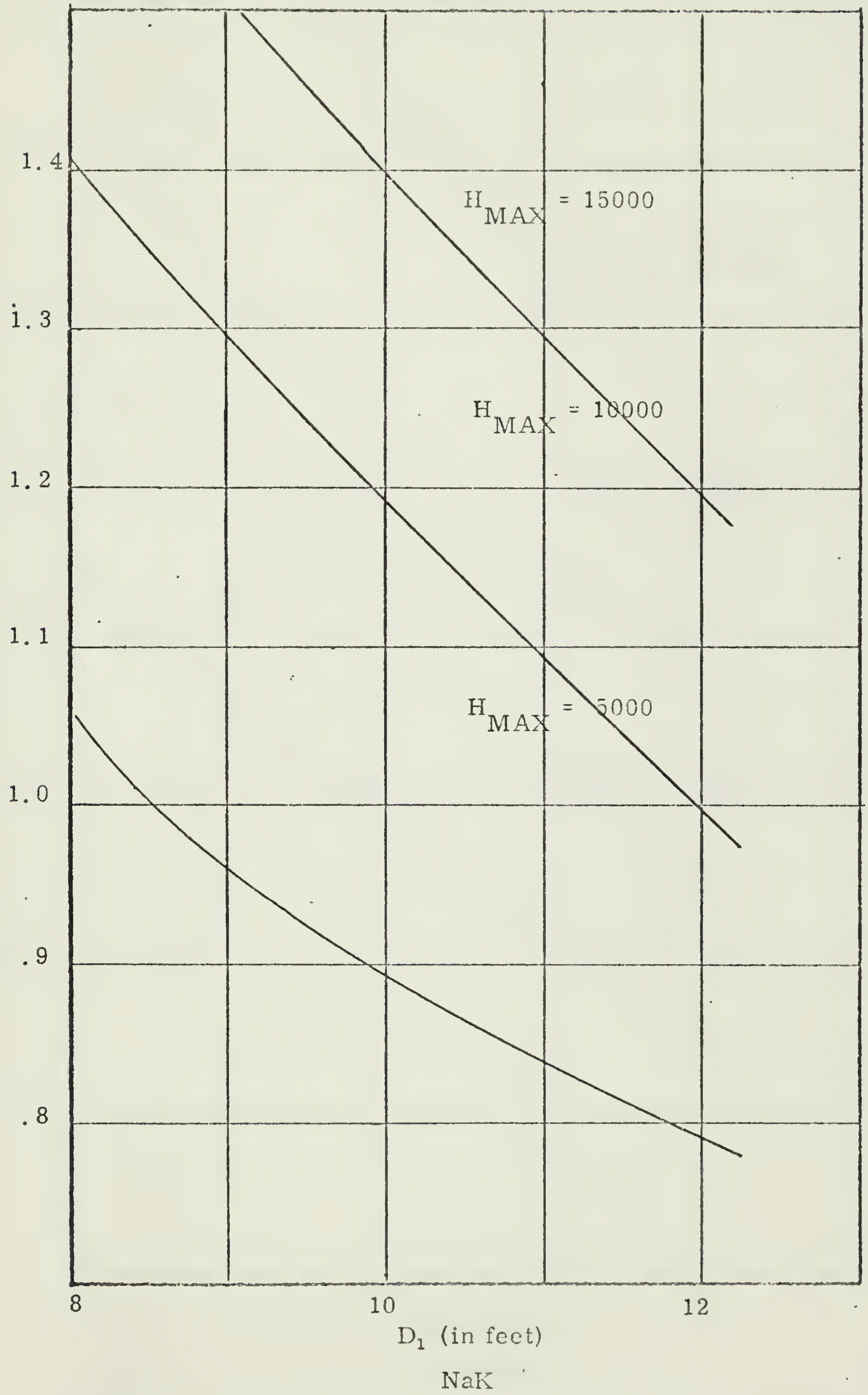


FIGURE (3.5) OPTIMUM TUBE DIAMETER

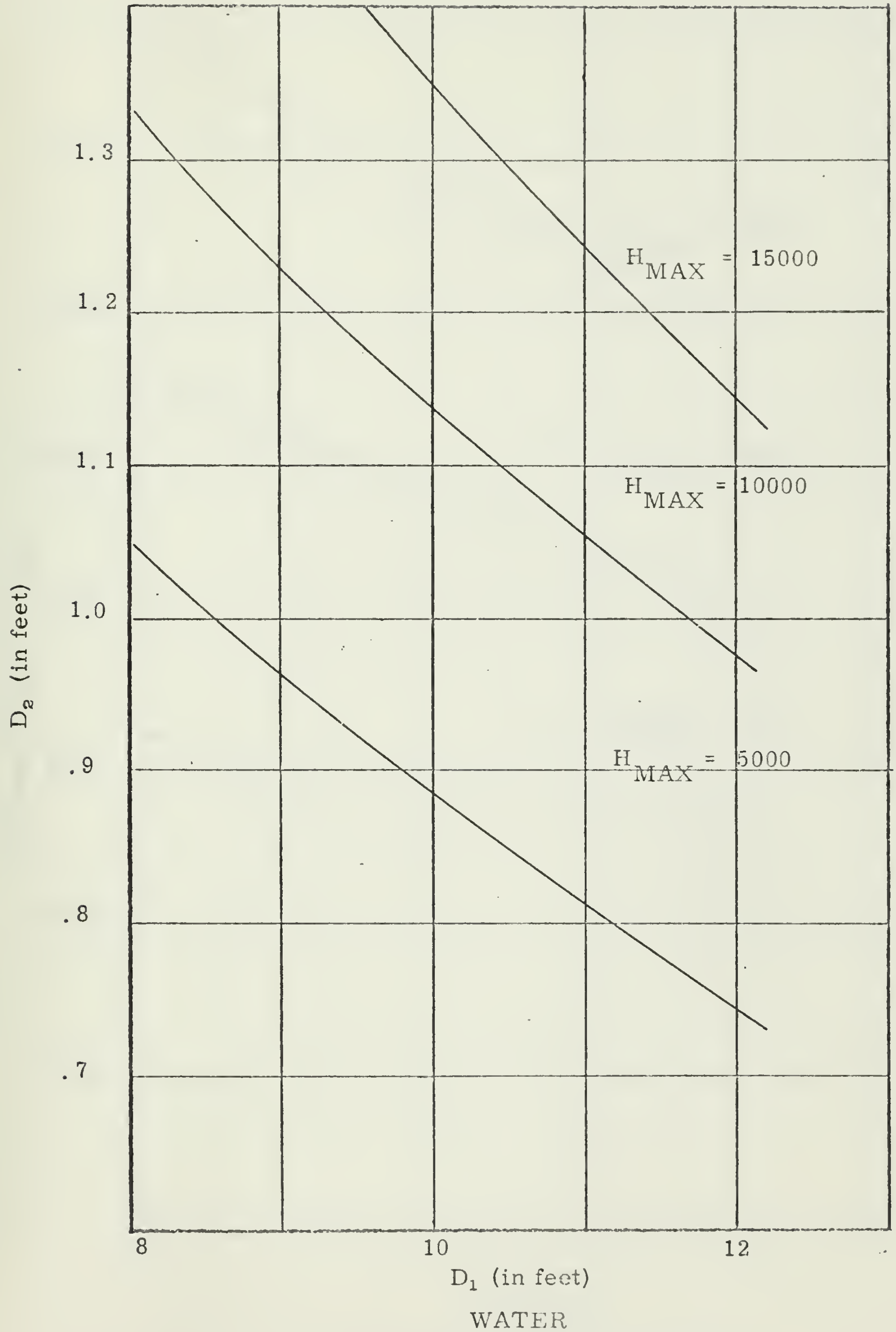
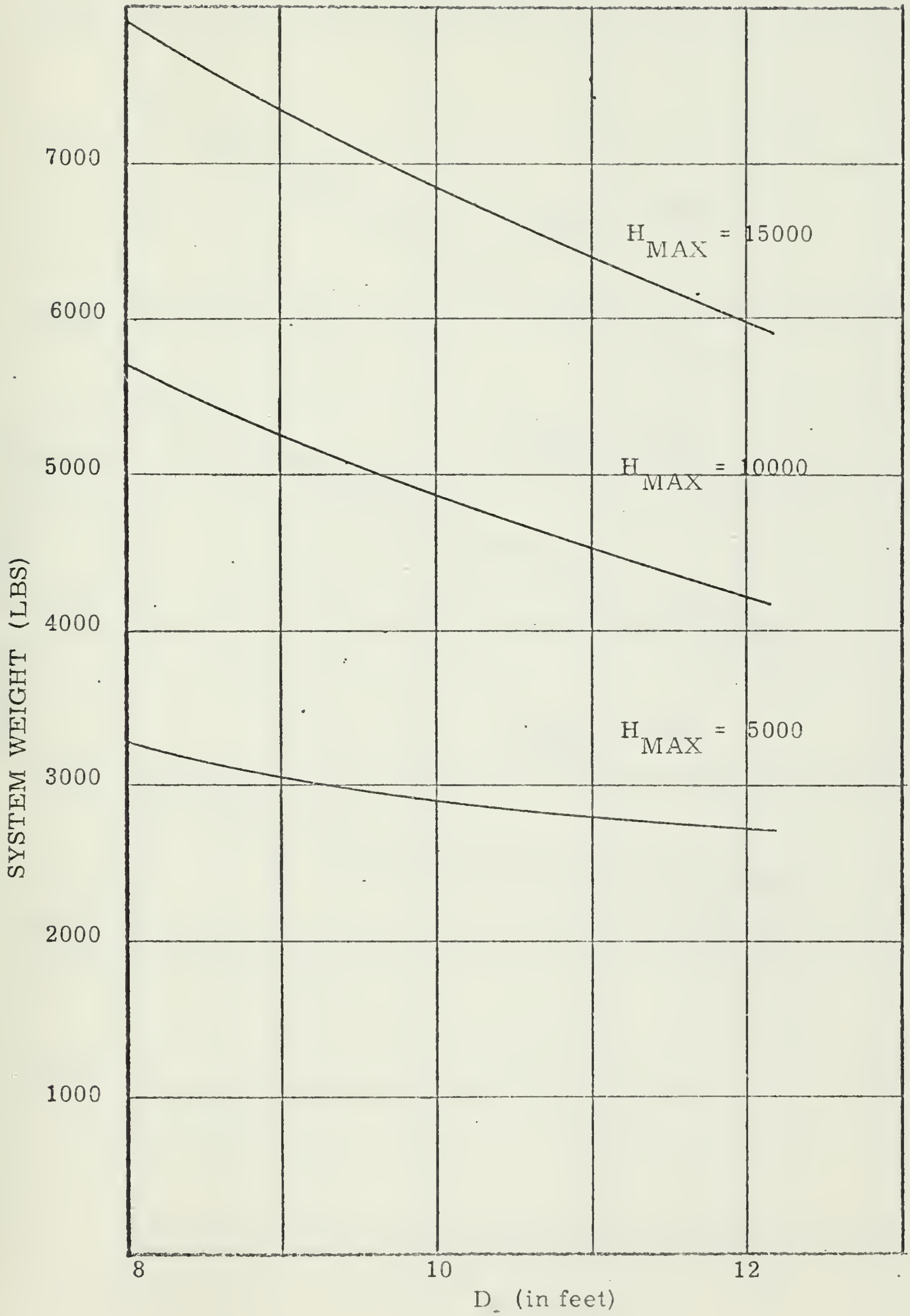
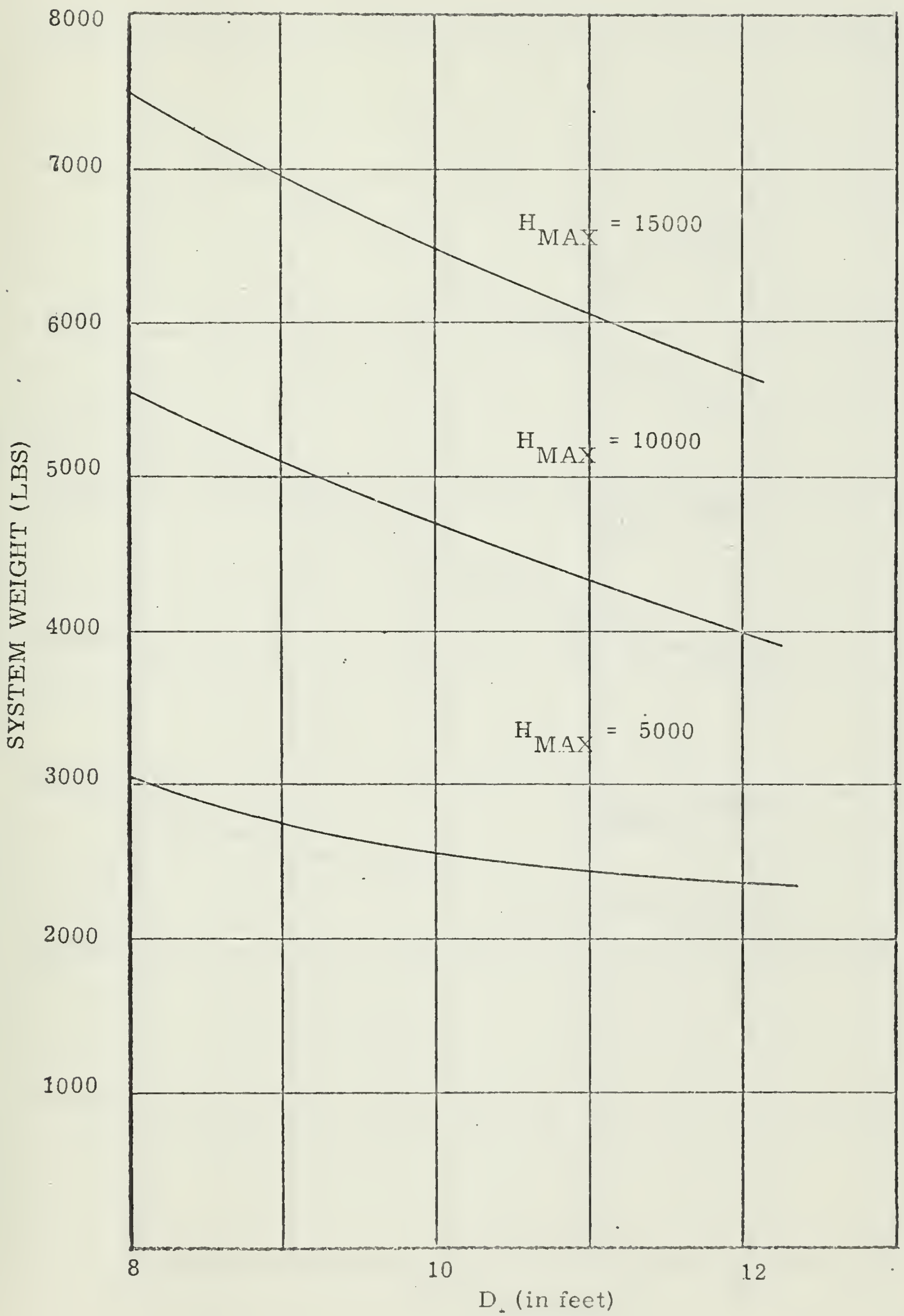


FIGURE (3.6) OPTIMUM TUBE DIAMETER



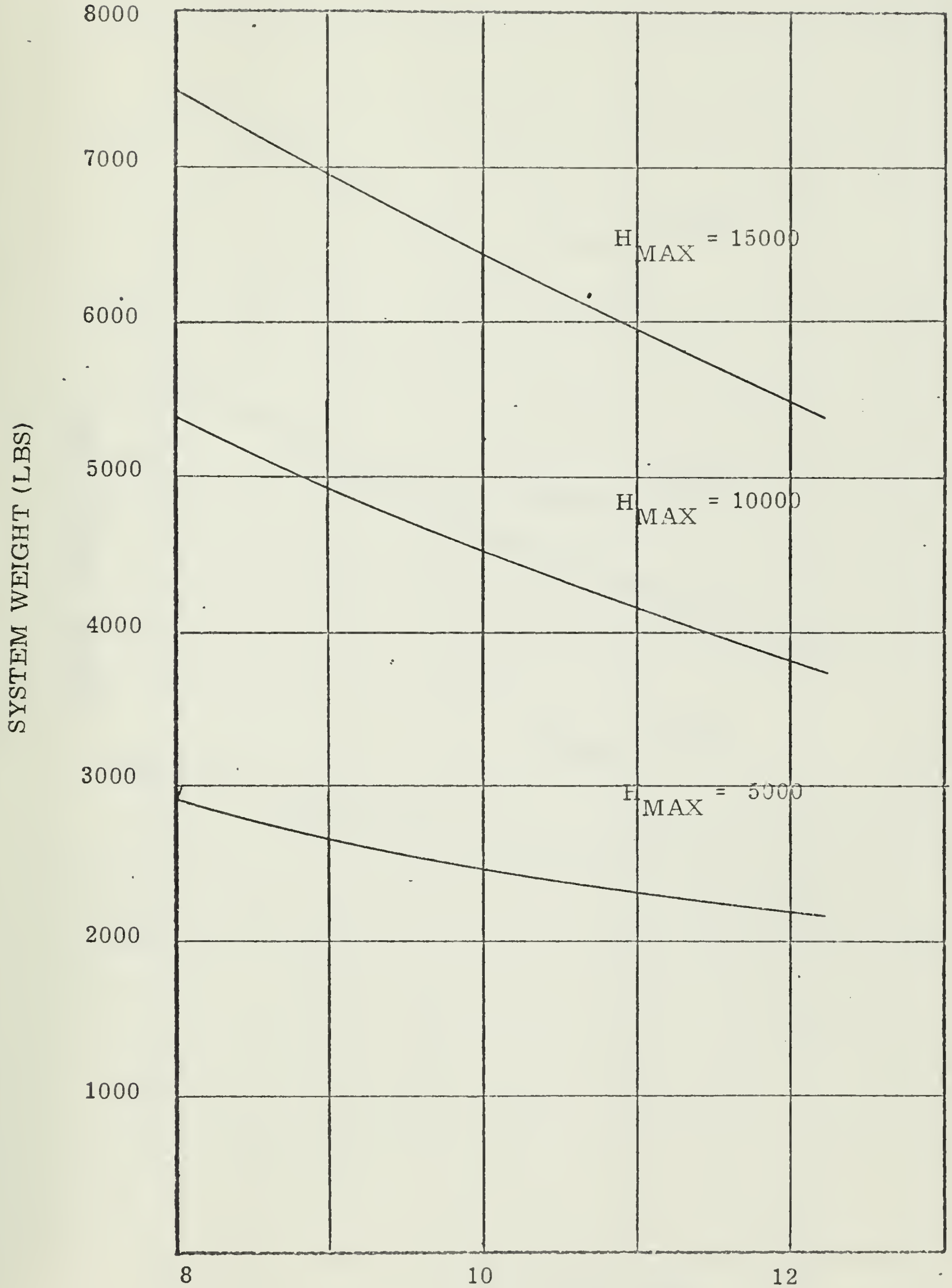
MERCURY

FIGURE (3.7) SYSTEM WEIGHT



BISMUTH

FIGURE (3. 8) SYSTEM WEIGHT



D_1 (in feet)

NaK

FIGURE (3.9) SYSTEM WEIGHT

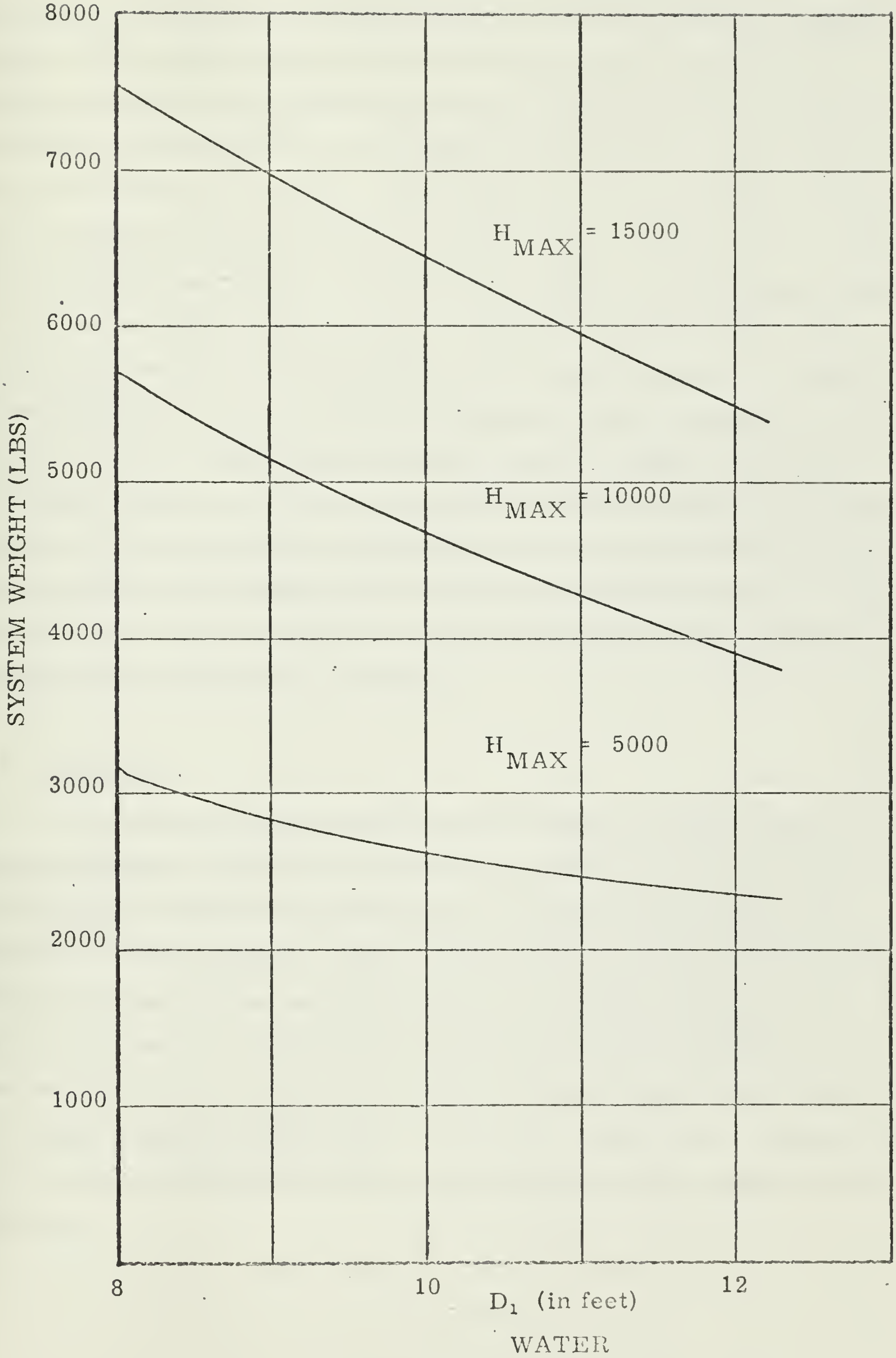


FIGURE (3. 10) SYSTEM WEIGHT

level and loop diameter that using NaK produced the minimum weight system. This occurs because the power loss at the maximum momentum storage level is proportional to the density and the viscosity of the fluid. NaK had the lowest density and viscosity of all the fluids used, hence the power supply weight was small. It is felt that actually more weight would be saved by using NaK than is indicated by the study because of the assumptions made for pump weight. At the optimum diameter for the NaK system the pump weight is a considerable portion of the total weight. This assumption was purposely taken to be large, however it is felt that a specially built pump for this application could be made lighter. This would indicate by itself that NaK is the best of the fluids considered, however the optimum tube diameters for the NaK systems were considerably larger than for the others. This would present a problem if the tubing were to be incorporated into the structure of the vehicle. If the tubing were to be run exterior to the vehicle the added volume would not be a problem.

3.5 Summary

As with any optimization study the results are only as good as the assumptions. However the important result here is not the determined system weights but the fact that a study like this results in a minimum value of system weight at a specific tube diameter. Knowing this tube diameter, the loop diameter, and the value for H_{MAX}, all other parameters in the system are determined. By taking the values obtained from this study the flow rate of the fluid can be determined, and a type of pump can be selected. Then the process can be repeated with more accurate assumptions and a truly optimum system weight can be attained.

CHAPTER 4

PUMPING DEVICES

4.1 General

The inherent problems associated with the space environment require special types of pumping devices for the fluid flywheel. The minor problems of pump maintenance here on earth such as replacing shaft packing, wear rings, or making adjustments become catastrophic problems in space. The normal leakage of most pumps through the shaft seals can not be tolerated in a space vehicle. Because of this special pumping techniques must be developed for the flywheel.

A survey of existing pumps was conducted to see if there were any pumps presently being manufactured that could be used in the fluid flywheel. The ideal pump for the flywheel would contain no moving parts, be efficient for all flow conditions, and be capable of providing large pressures instantaneously. At the present time no pumps exist which satisfy these conditions completely, however the following types of pumps appear to have possibilities for future development.

4.2 Electromechanical Pumps

The electromechanical pump utilizes the motor principle to pump fluid. The fluid, which must be a conductor, passes through a magnetic field while conducting a current. The interaction of the field produced by the flow of current through the fluid and the magnetic field produces forces acting on the fluid. This in turn accelerates the fluid and flow results. These pumps can operate on alternating or direct

current, they have no moving parts, and present no leakage problems. The disadvantages of this type are its relatively low efficiency and high weight. However, at present they seem to be the most adaptable to the fluid flywheel.

4.3 Mechanical Pumping Devices

There are numerous types of mechanical pumps but they all have several moving parts and almost always have a leakage problem. There are several possible ways of eliminating the leakage problem. First, placing the pump motor inside the pump will eliminate the shaft sealing problem. The power for the motor would be supplied by conductors going through the casing; because the conductors are not moving they can be sealed easily. Pumps like this are presently manufactured and could be used in the flywheel. The disadvantages of this type of pump are that the losses are higher due to fluid drag on the motor and that there are moving parts which increase wear, decreasing the reliability.

Another possibility is having the impeller of the pump serve the dual purpose of pumping the fluid and acting as the rotor of the pump motor. The impeller could be permanently magnetized and driven by a field through the casing in a manner similar to brushless D. C. motors. Harmonic drive devices could also be used to drive the impeller through the casing. Pumps of this type would have no leakage problem because there are no mechanical parts penetrating the casing, and they have fewer moving parts than conventional designs. Their reliability would be almost as good as the electromagnetic pumps and their efficiency would probably be much higher.

4.4 Summary

The problem of providing a suitable pumping device for fluid flywheels is a problem common with all flueric systems. One of the

advantages of flueric systems is that there are no moving parts and because of this almost no wear. At the present time there are no suitable flueric power supplies, and therefore these systems are usually powered by mechanical pumps or compressors. There is a great deal of research being conducted in component design for flueric systems and it is felt that it will just be a matter of time before a suitable pump will be developed.

CHAPTER 5

FLUID FLYWHEEL CONTROL SYSTEM

5.1 Analysis of System

The flywheel is an accelerating device which applies a torque on the vehicle as long as the fluid velocity is changing. The torque applied to the vehicle can be analyzed by referring to Figure (5.1). Due to the movement of the fluid in the pipe, local shear stresses are set up. The sum of these shear stresses times their moment arm produces a torque opposite to the direction of the fluid movement. This torque acts as a damping torque and produces the system's only natural damping.

The pressure drop across the pump exerts a torque equal to the pressure times the area multiplied by its moment arm. The difference between these torques is the torque that acts to accelerate the vehicle.

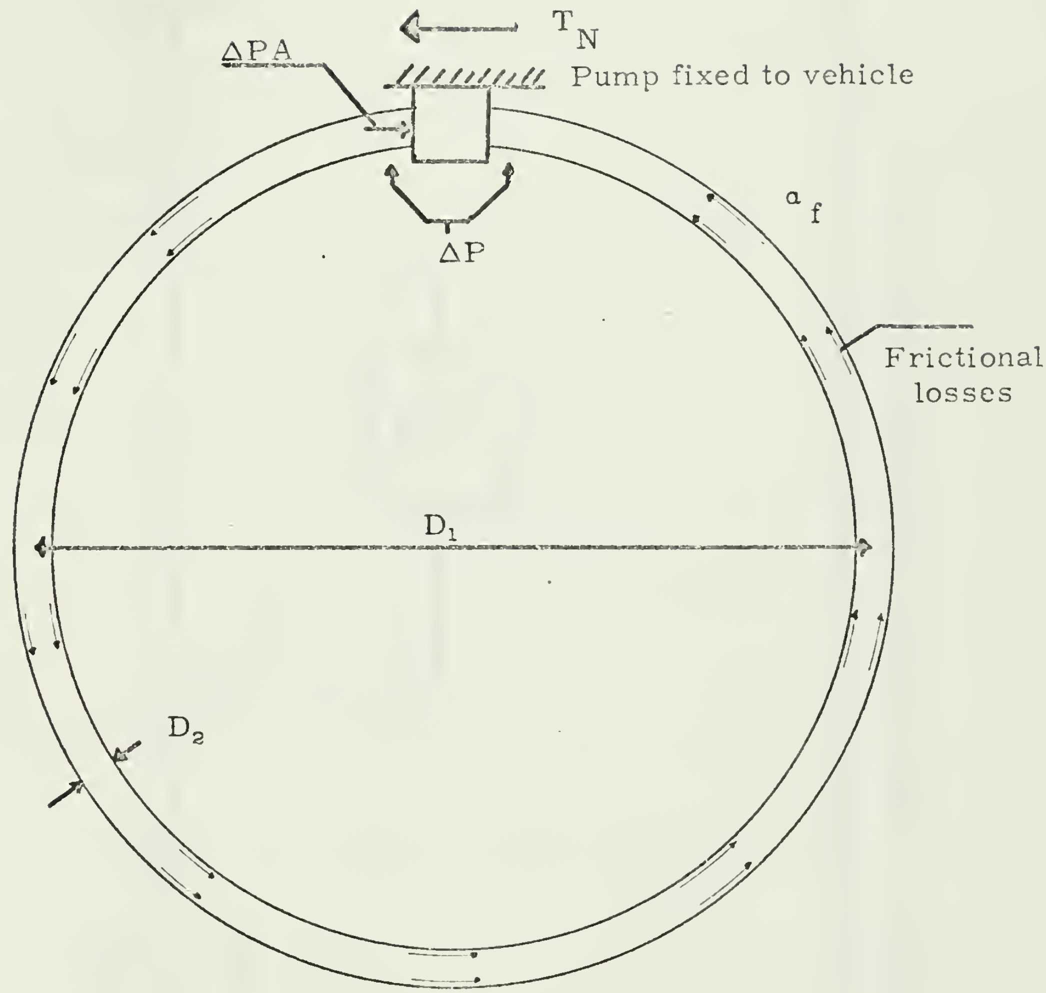
$$T_N = T_p - T_L \tag{5.1}$$

$$a_v = \frac{T_N}{I_v} \tag{5.2}$$

This same net torque acts on the fluid. Therefore the fluid's acceleration is:

$$a_f = \frac{T_N}{I_f} \tag{5.3}$$

The system can be represented as shown in Figure (5.2) where K is the linearized damping constant and takes the frictional losses in the loop into account. T_D is disturbance torques.



T_L = Sum of frictional forces times $D_1/2$

T_p = Pump torque

FIGURE (5.1) TORQUE ACTING ON VEHICLE

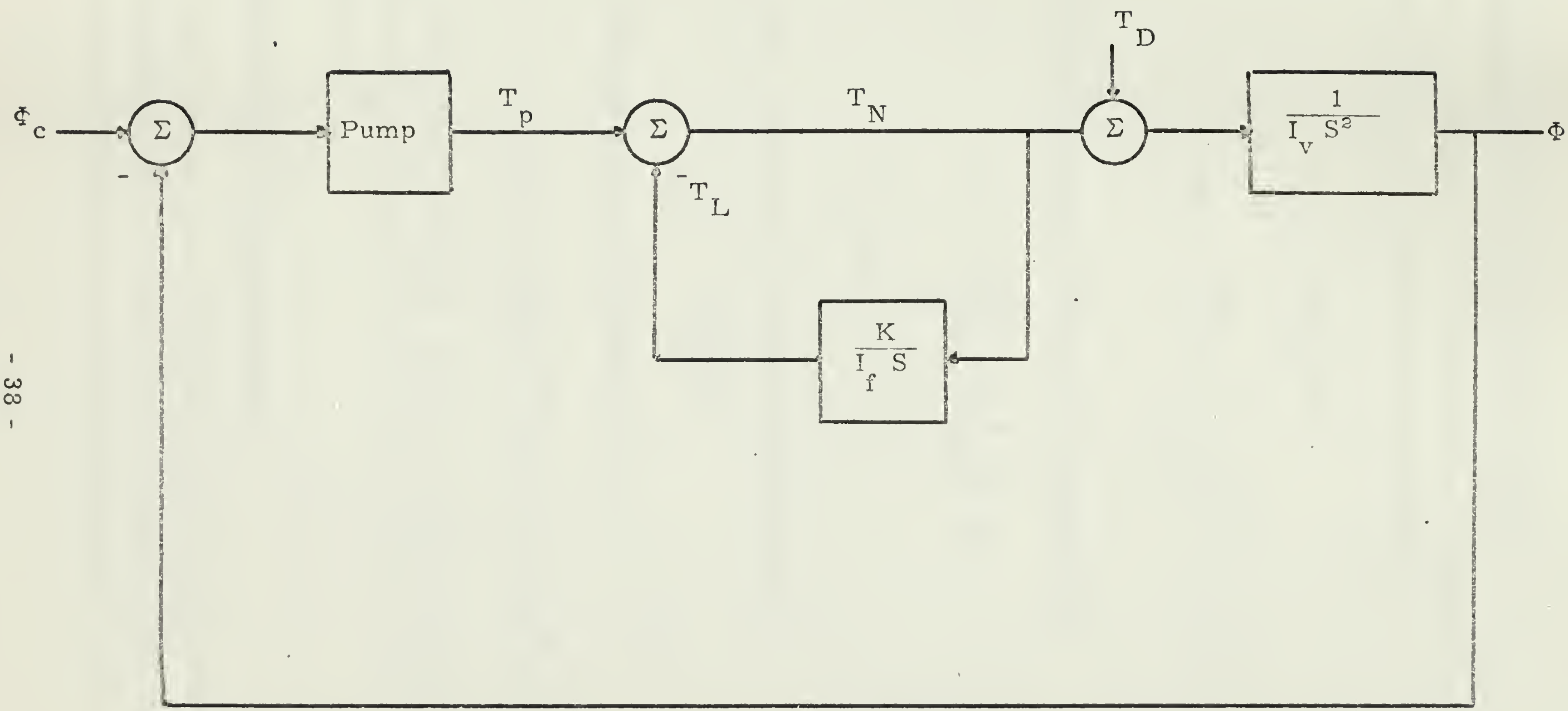


FIGURE (5.2) BASIC SYSTEM WITH UNITY FEEDBACK

5.2 Evaluation of Linearized Damping Constant

The losses in the loop produce a torque equal to the pressure drop times the area multiplied by the moment arm.

$$T_L = \frac{\Delta p \, \pi \, D_1 \, D_2^2}{8} \qquad (5.4)$$

$$\Delta p = \frac{.0483 \, \rho_f^{.84} \, D_1^{2.84} \, \mu^{.16} \, \omega_f^{1.84}}{D_2^{1.16}} \qquad (A\ 4)$$

The block diagram (Figure (5.2)) requires that:

$$T_L = K \frac{T_N}{I_f s} = K \int_c^t \frac{T_N}{I_f} \, dt \qquad (5.5)$$

$$T_K = K \, \omega_f \qquad (5.6)$$

Setting Equation (5.4) equal to Equation (5.6):

$$K \, \omega_f = \frac{\pi \, D_1 \, D_2^2 \, \Delta p}{8}$$

Substituting for Δp and solving for K:

$$K = .006 \, \pi \, \rho_f^{.84} \, D_1^{3.84} \, \mu^{.16} D_2^{.84} \, \omega_f^{.84}$$

This would be somewhat different if there were any diffusers or restrictions in the pump or in the loop, however it would still be a function of $\omega_f^{.84}$.

5.3 Determination of System Response

The system as shown in Figure (5.2) is similar to the one proposed in Reference (1). This system was actually built and tested by the authors of Reference (1). Although the system works, there are

two problems associated with it. First, the system response varies greatly with the amount of stored momentum. A command in angle change with the fluid at rest has a much less damped response than the same command when the wheel is operating in the momentum storage mode and K is higher. Second, an impulse in disturbance torque creates a steady state position error.

Because of these two reasons, the system would hardly be adequate for manned missions. The reason for having the system is to maintain position and if every small disturbance torque would move the vehicle to a new position it would be unsatisfactory.

An integrating circuit was added to the system to eliminate the steady state error, however now the system becomes third order and the inherently stable qualities of second order systems are gone. In order to compensate for this, velocity feedback can be added to the system. This results in a modified block diagram as shown in Figure (5.3). The closed loop transfer function for this system is:

$$\frac{\Phi}{\Phi_c} = \frac{G I_f (s + \frac{1}{\tau})}{I_v I_f s^3 + (K I_v + G I_f C) s^2 + (\frac{I_f G C}{\tau} + G I_f) s + \frac{G I_f}{\tau}} \qquad \text{(D 6)}$$

This expression is derived in Appendix D.

Applying Routh's Stability Criteria:

$$A s^3 + B s^2 + C s + D$$

$$\begin{array}{lll} s^3 & A & C \\ s^2 & B & D \\ s^1 & \frac{BC - AD}{B} & \end{array}$$

Since the signs of all the coefficients are positive there will be no poles in the right half of the s-plane if the product BC is greater than AD. Expressing this in terms of the system parameters:

$$(KI_v + GI_f C)\left(\frac{I_f GC}{\tau} + GI_f\right) > (I_v I_f)\left(\frac{GI_f}{\tau}\right) \qquad (5.7)$$

From Equation (5.7) it is obvious that if a large enough gain for C is taken the system will be stable. However, if C is too large the system will be overly damped and the response will be very slow. It was therefore determined to use a large value for C when K was small and to decrease C as K increased. With the aid of Reference (10) the appropriate values for C and the gain of the pump (G) can be selected to give the desired response.

By using a multiplicative feedback system C can be varied as K changes to keep the response relatively constant for different initial fluid velocities. In this manner the response to a step input while the system is operating in a low momentum storage mode will look almost the same as when the system is operating in a high momentum storage mode.

Suppose it was determined to keep the coefficient of the s^2 term in Equation (D 6) equal to a constant (B).

Then:

$$C = \frac{B - KI_v}{GI_f} \qquad (5.8)$$

In this manner C can be determined to be a function of fluid velocity.

$$C = \frac{B}{GI_f} - \frac{I_v}{GI_f}(.006)\,\pi\,\rho_f^{.84}\,D_1^{3.84}\,\mu^{.16}\,D_2^{.84}\,\omega_f^{.84}$$

The value of ω_f at any particular time could be obtained by measuring the pump speed if a positive displacement pump is used. For other types of pumps determining the fluid velocity would be more difficult, however, mechanical means could always be resorted to. At any rate there is some realizable means of measuring fluid velocity which can be used in the multiplicative feedback system shown in Figure (5.4)

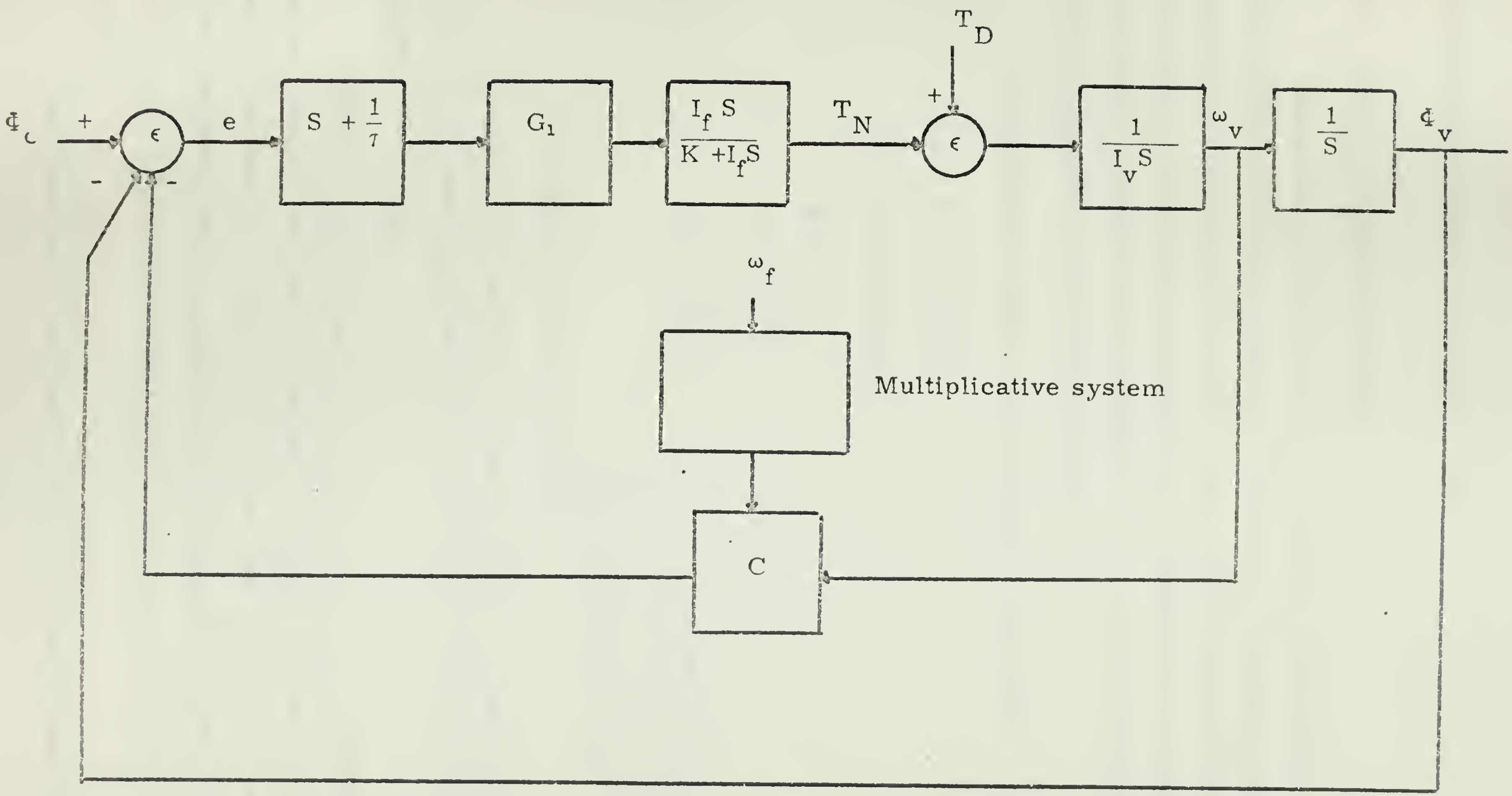


FIGURE (5.3) SYSTEM WITH COMPENSATION

to determine the appropriate value of C.

5.4 Analog Computer Simulation

Because time did not permit, a model of this system was not built. However the General Electric Advanced Technology Laboratories have constructed a satellite model with a fluid flywheel control system. The results of their tests are published in Reference (1). It was felt that roughly the same input parameters should be used in this simulation so that some comparisons could be drawn.

The following values were assumed for this study:

I_v = 120 ft-lb sec^2

I_f = 0.935 ft-lb sec^2

D_1 = 4 ft

D_2 = 0.03 ft

G = 40 ft-lb/rad — Assumed constant over the entire range. Overall pump, power conditioning and sensor gain.

Mercury - the operating fluid.

K was approximated as a linear function of ω_f and is shown in Figure (5.5) assuming there were no other losses than the frictional losses in the tubing.

These values uniquely determined two of the coefficients in the denominator of the transfer function. The other two were determined using Reference (10). Figure (5.6) shows how the time constant, natural frequency, and damping ratios change as the values of K and C are varied. Figure (5.7) shows how C varies as a function of fluid velocity.

The linearized model was placed on an analog computer to

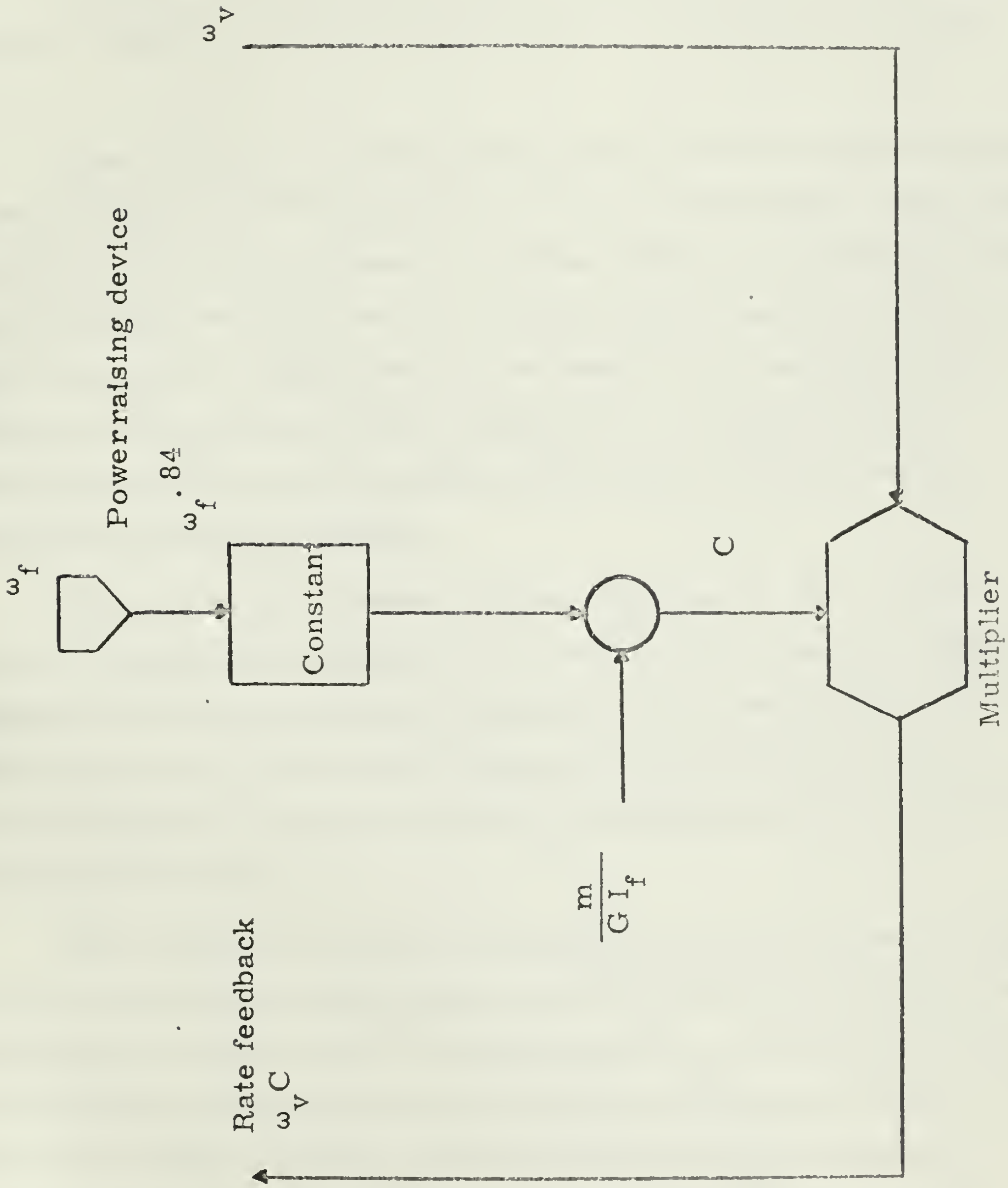


FIGURE (5.4) MULTIPLICATIVE FEEDBACK SYSTEM

shown in Figure (5.8). Both amplitude and time scaling were used to keep the response within the limits of the computer.

It was found that the values originally chosen for C were a little too large and the responses were overdamped. C was therefore decreased slightly. The new values for C are also shown in Figure (5.7).

An oscillograph recorder was used to plot the results which are shown in Figures (5.9 through 5.19). These are the actual plots of the recorder, not approximations or tracings. Figures (5.9 through 5.14) show the linearized responses to a step in angle change for different initial momentum storage levels. The beauty of the multiplicative system is really shown by these responses. Although the natural dampening in the system changes by more than a factor of ten the responses are almost identical.

The next two responses, Figures (5.15 and 5.16), show the effect of varying the pump gain. In an actual system the pump gain may change for different momentum storage levels, however this was not taken into account in this study because no particular pump was selected. Figure (5.17) shows the effect of changing the time constant of the integrator circuit.

The response to a step in disturbance torque is shown in Figure (5.18). This shows a steady state error; the reader is cautioned that in reality this is not the case. If a step disturbance were applied eventually the system would saturate, stop producing torque, and then the vehicle would rotate. However, the disturbances will be impulses or short pulses and since the analysis shows that there is a constant steady state error for a step, there will be no steady state error for an impulse. The response to a short pulse is shown in Figure (5.19).

The response time appears to be long but the momentum storage capability of this system was very small. The moment of inertia of the

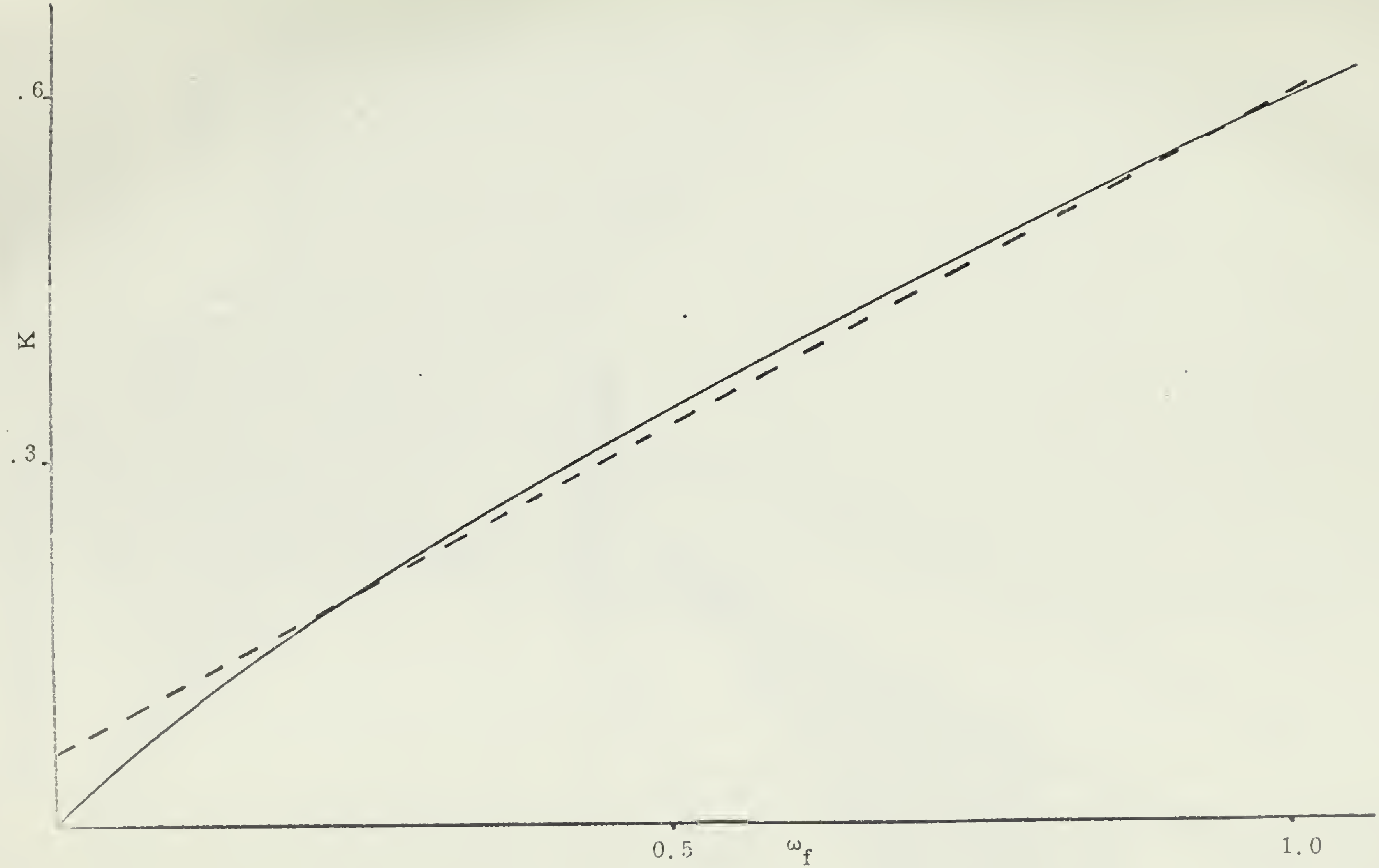


FIGURE (5.5) FRICTIONAL FEEDBACK CONSTANT AS A FUNCTION OF ω_f

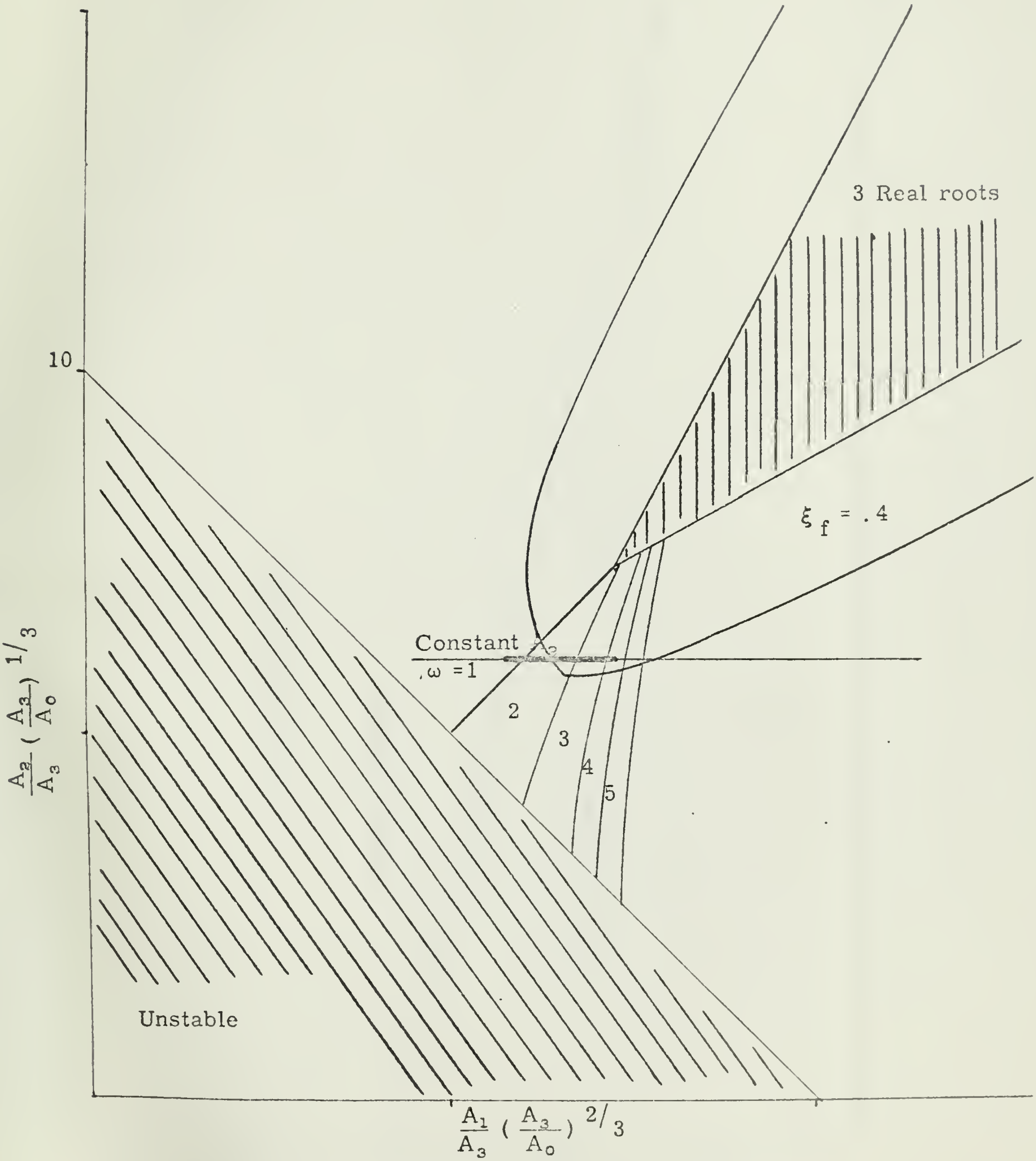


FIGURE (5.6) RESPONSE PARAMETERS

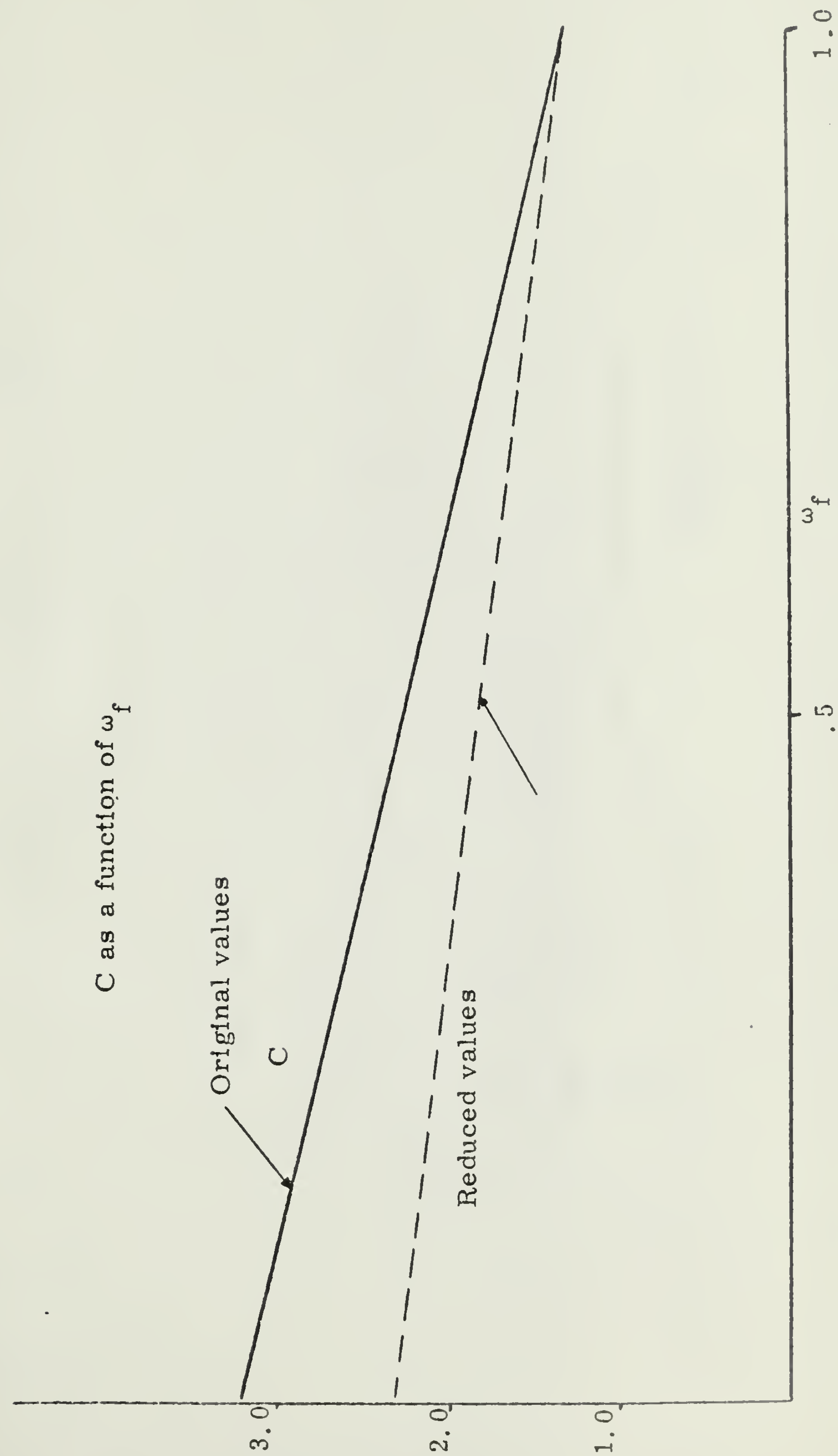


FIGURE (5.7) C AS A FUNCTION OF ω_f

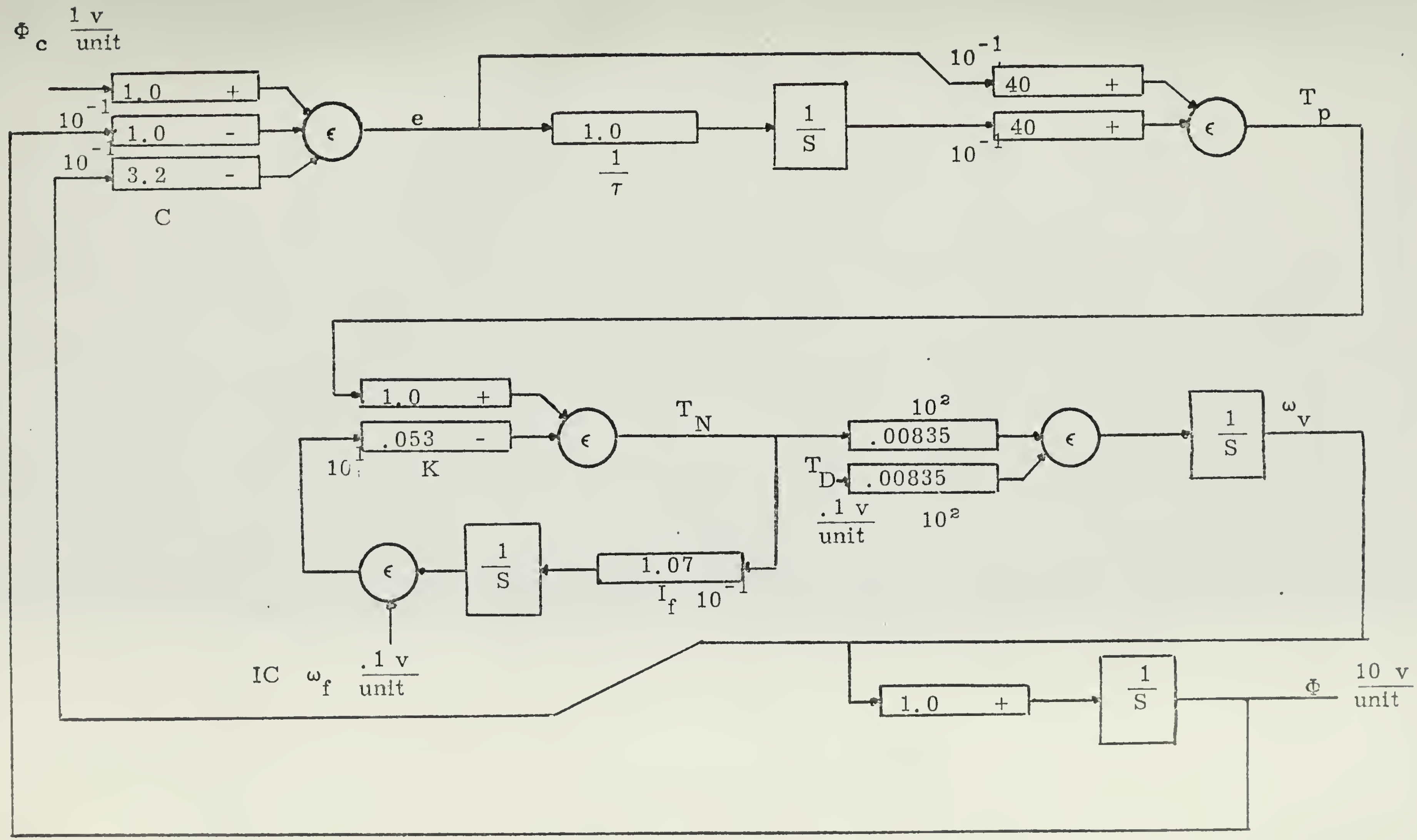


FIGURE (5.8) ANALOG COMPUTER PROGRAM

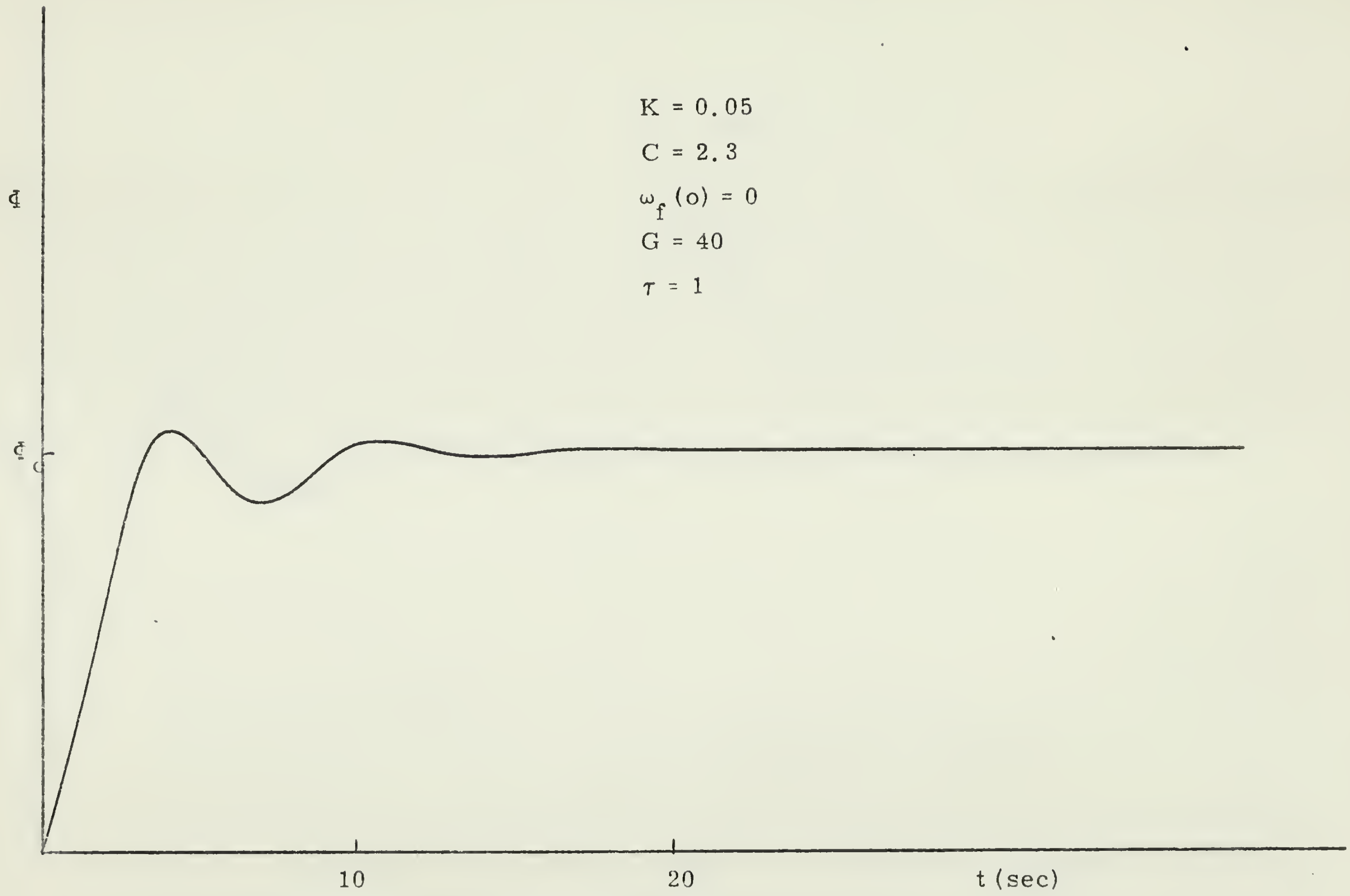


FIGURE (5.9) RESPONSE TO STEP IN Φ_c

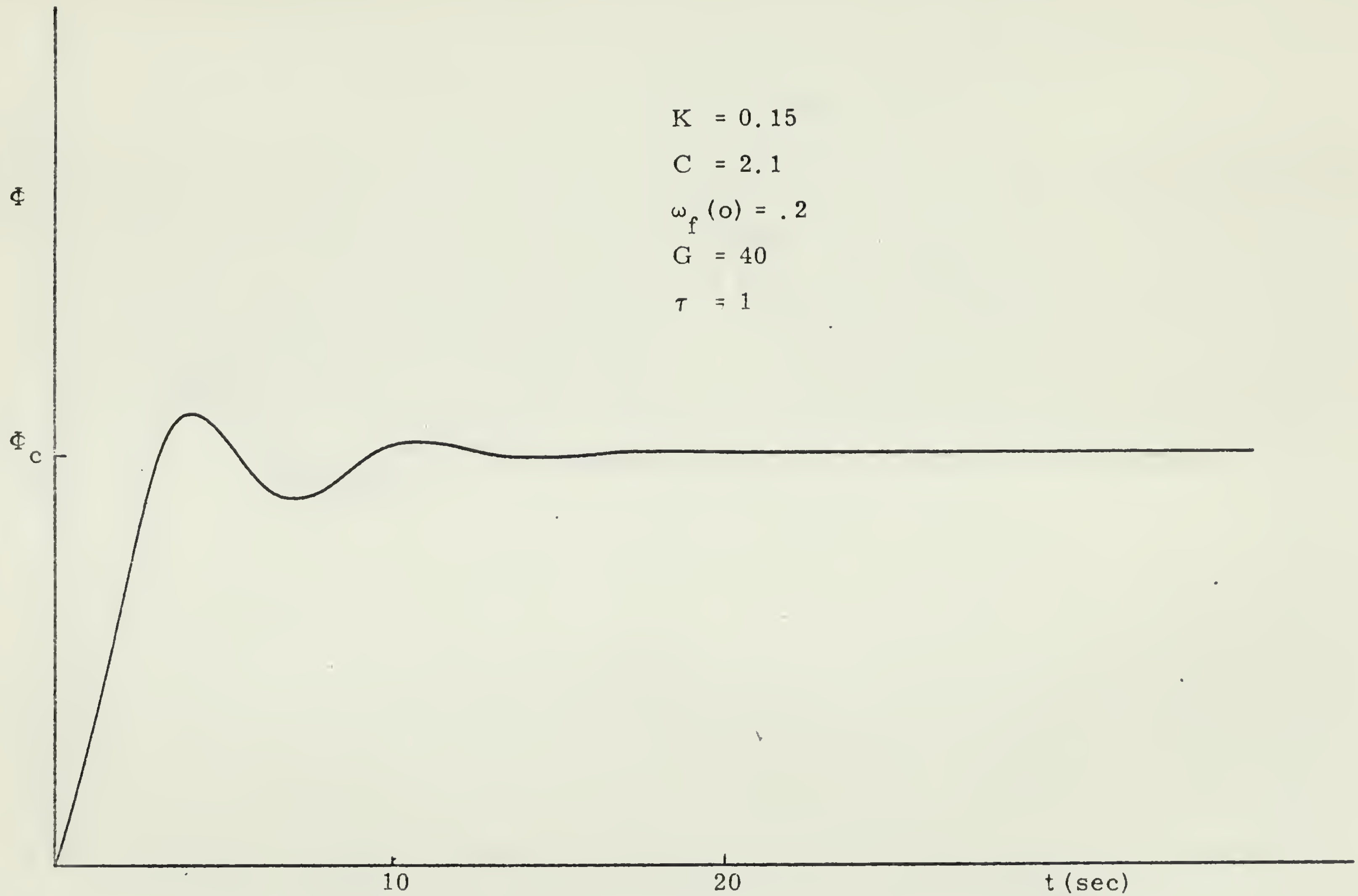


FIGURE (5.10) RESPONSE TO STEP IN Φ_c

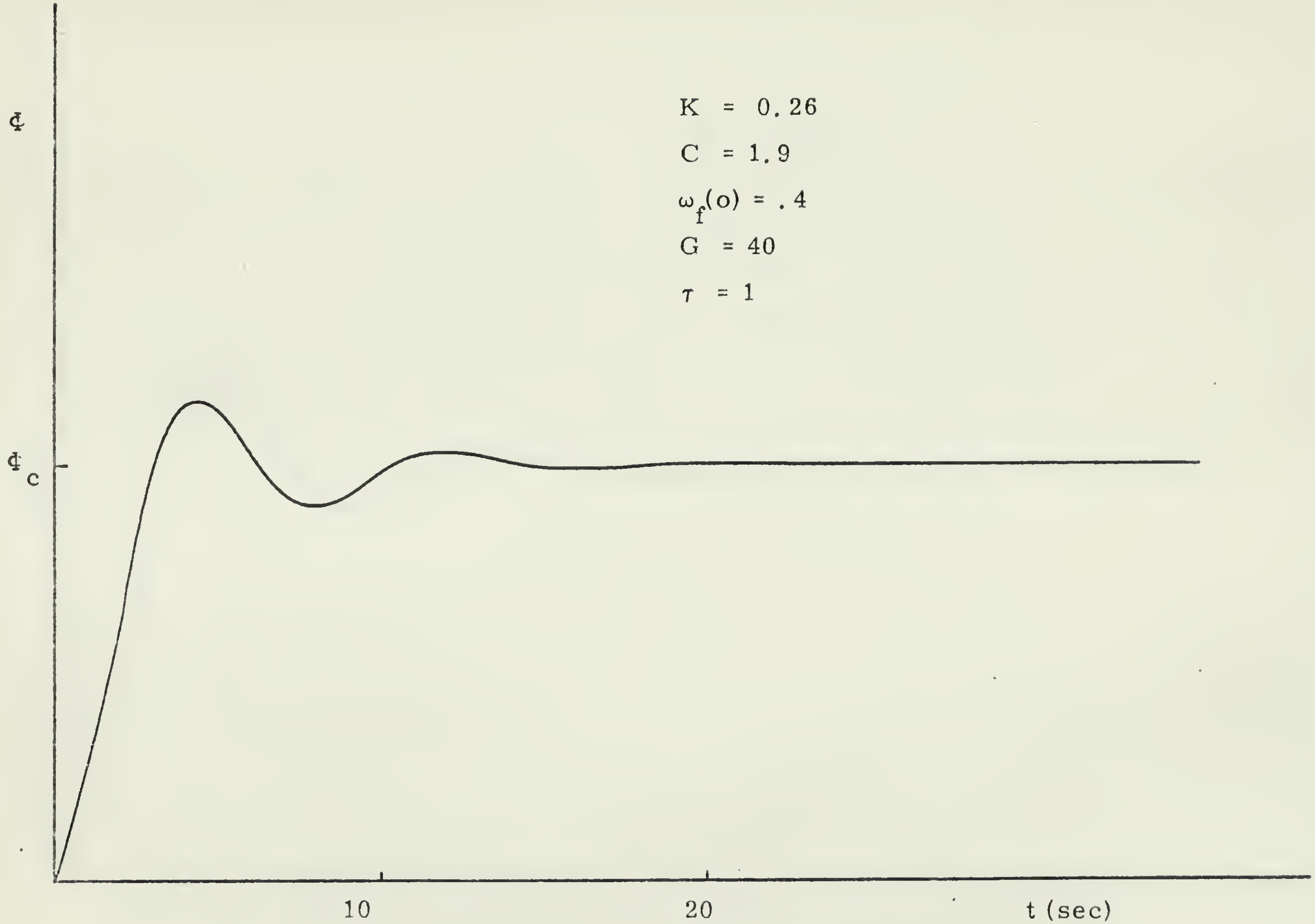


FIGURE (5.11) RESPONSE TO STEP IN Φ_c

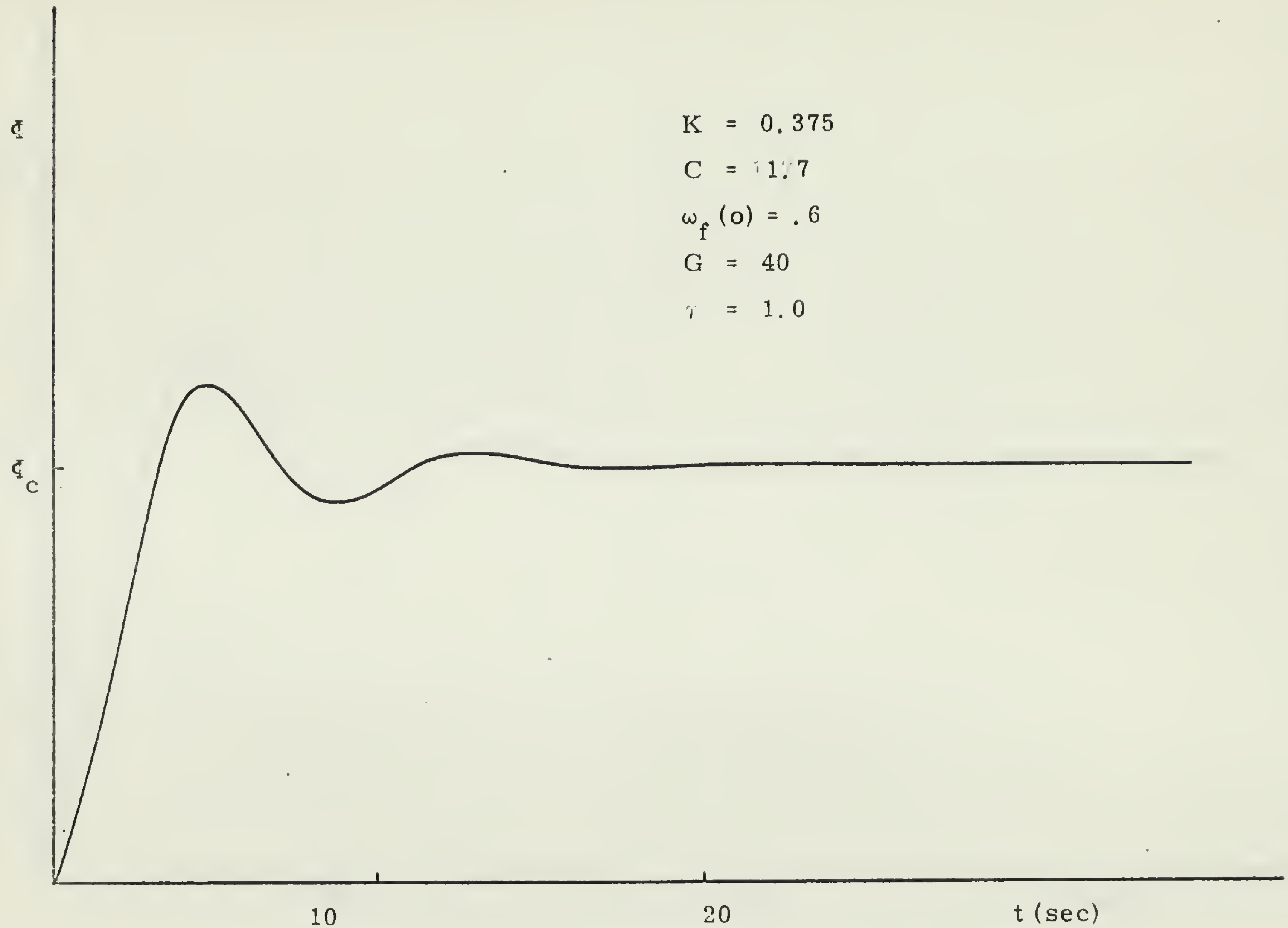


FIGURE (5.12) RESPONSE TO STEP IN $¢_c$

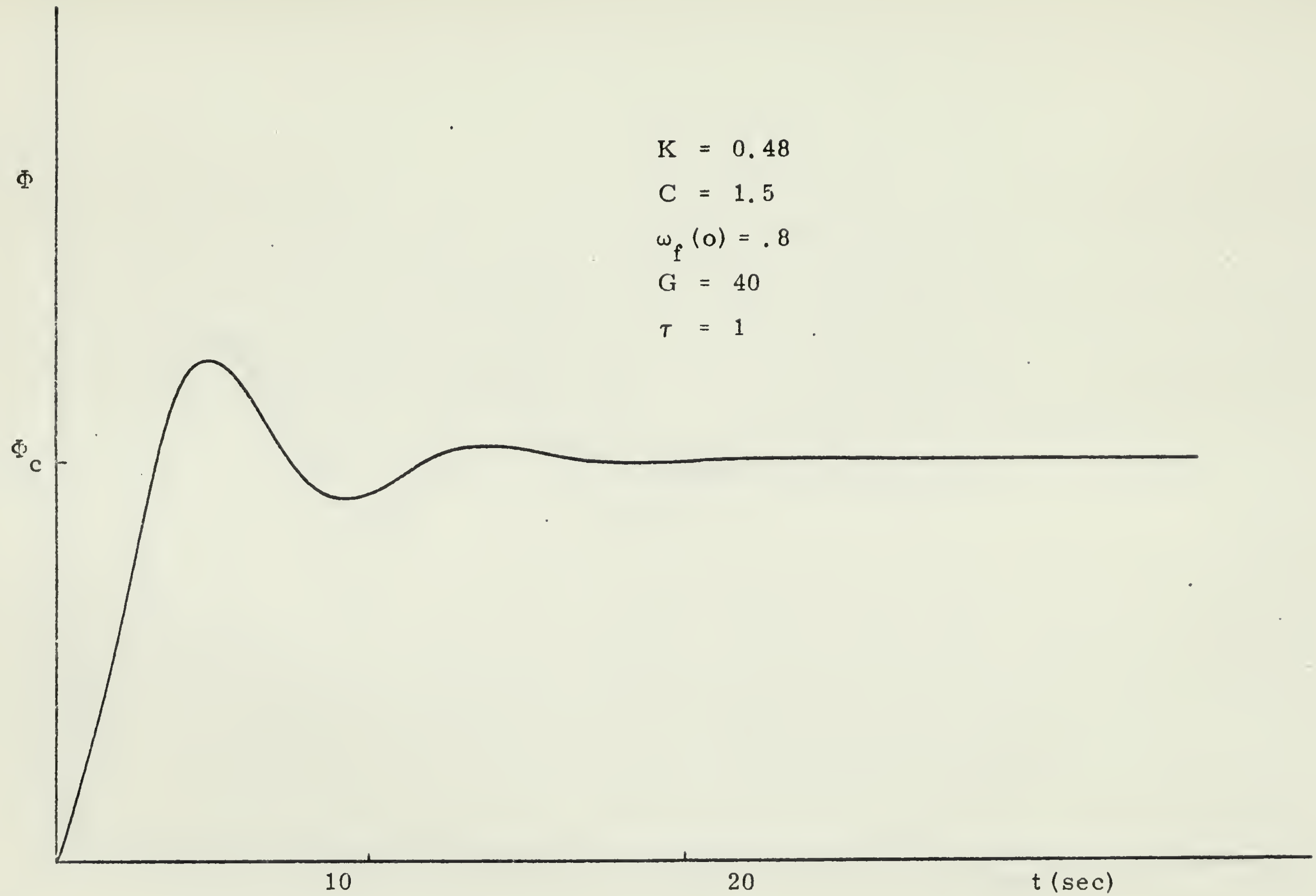


FIGURE (5.13) RESPONSE TO STEP IN Φ_c

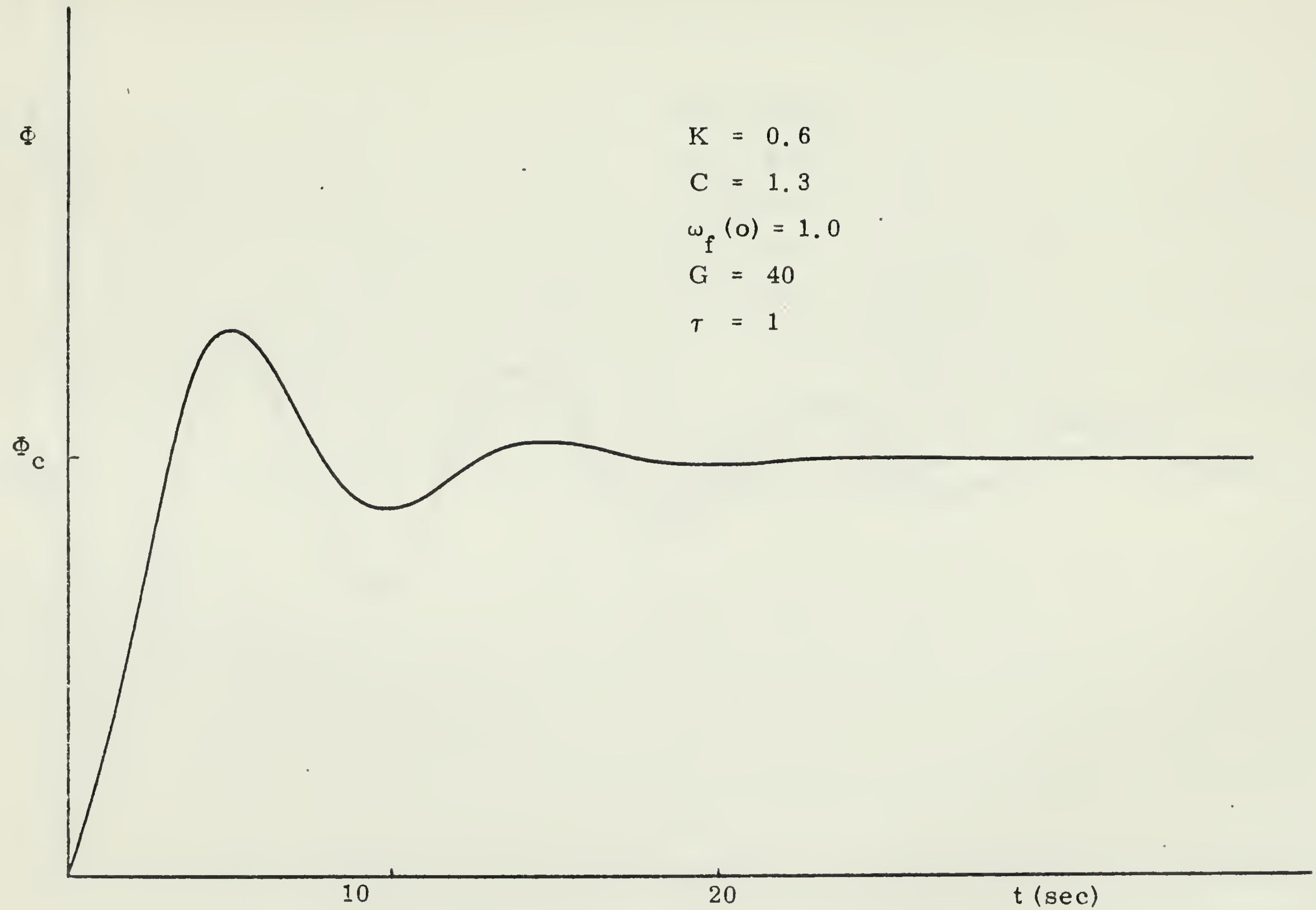


FIGURE (5.14) RESPONSE TO STEP IN Φ_c

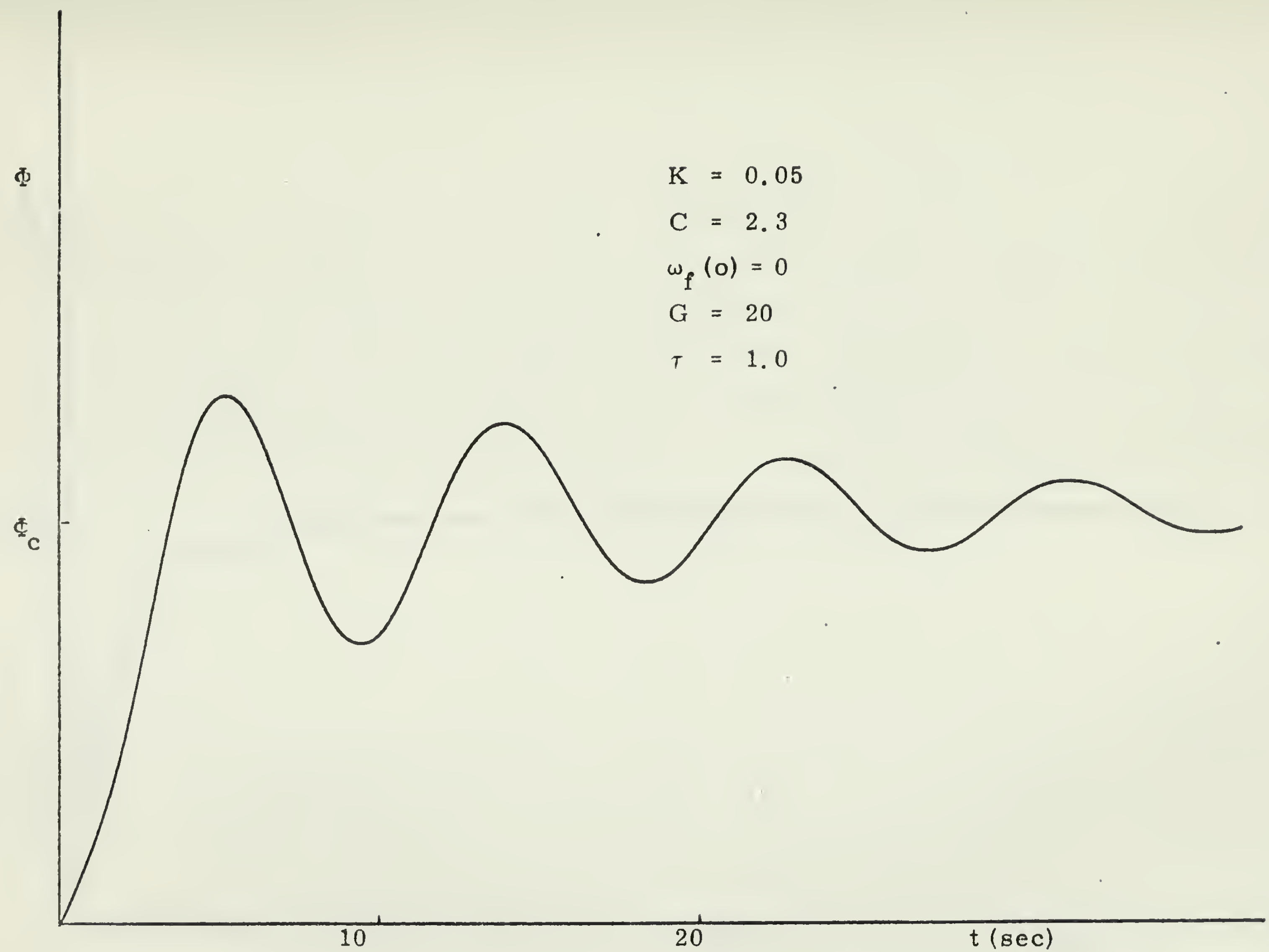


FIGURE (5.15) RESPONSE TO STEP IN Φ_c

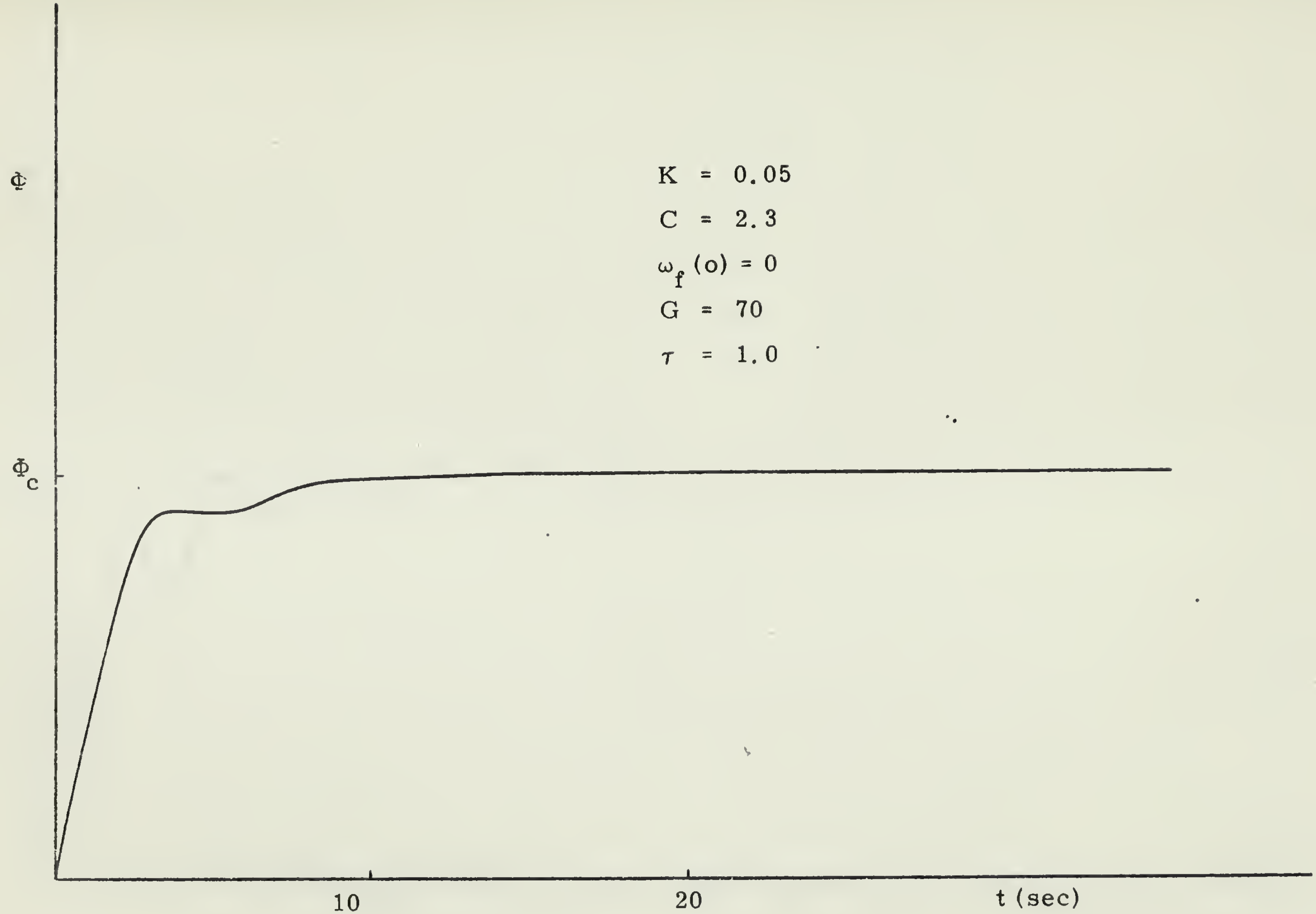


FIGURE (5.16) RESPONSE TO STEP IN Φ_c

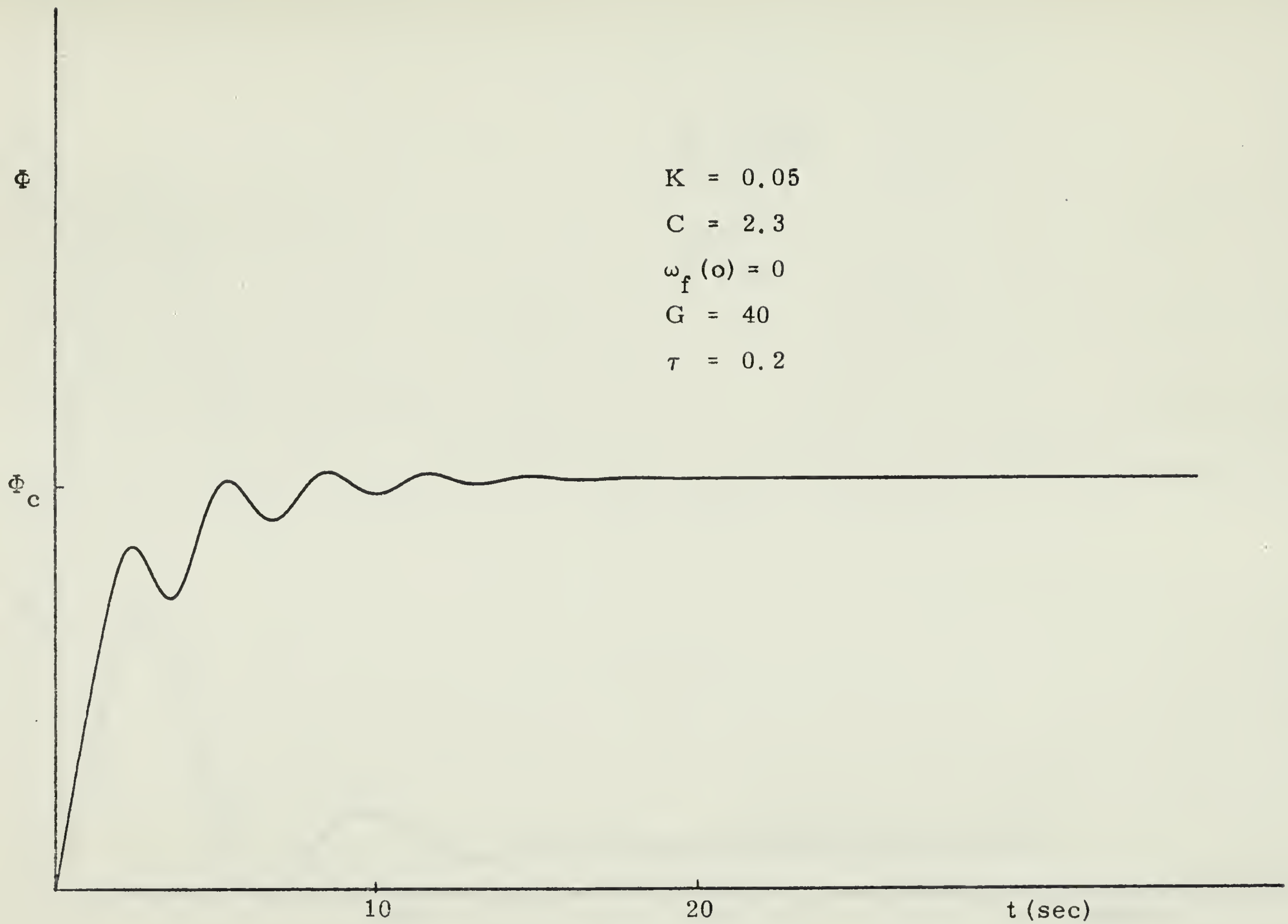


FIGURE (5.17) RESPONSE TO STEP IN Φ_c

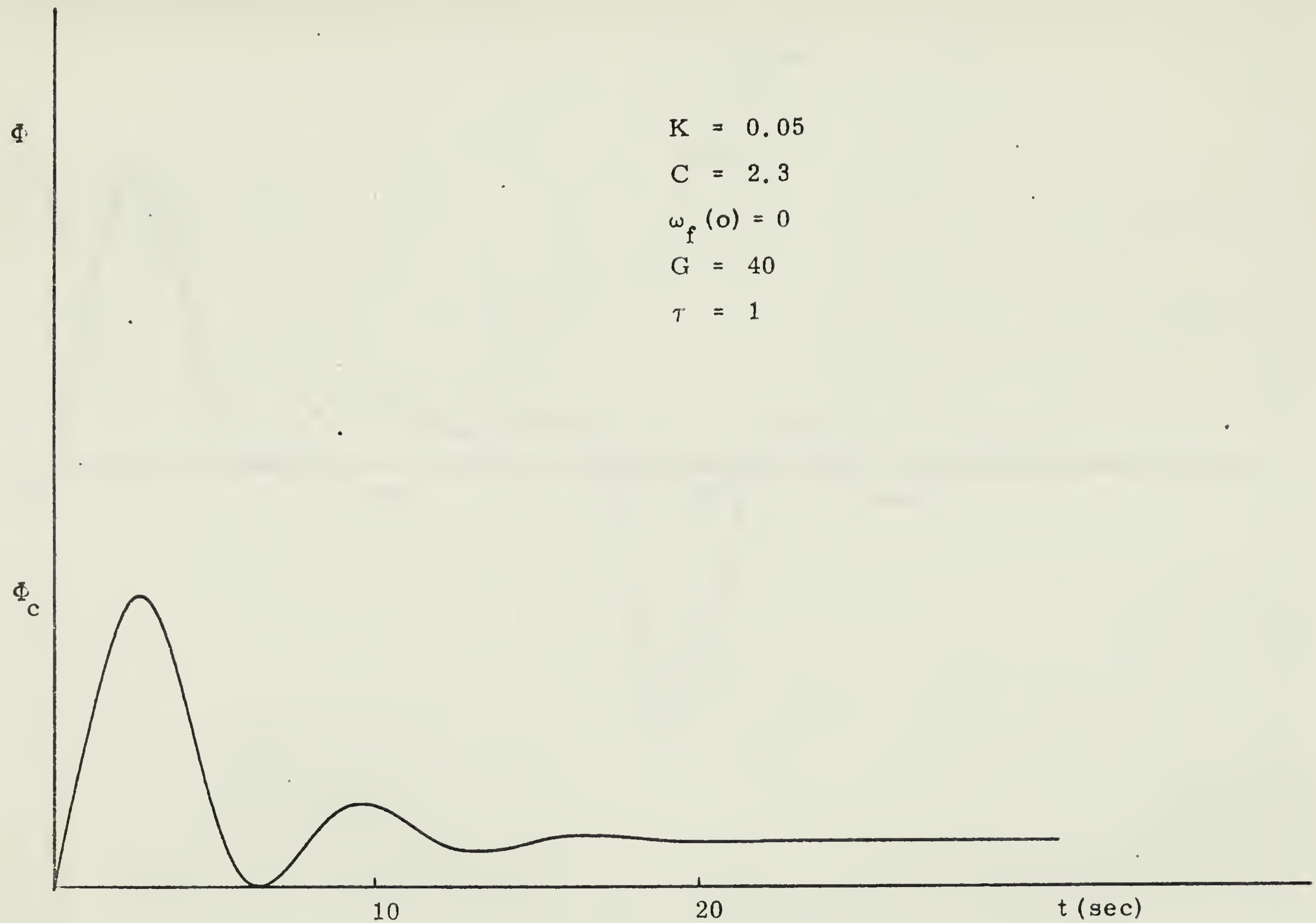


FIGURE (5.18) RESPONSE TO DISTURBANCE TORQUE (STEP)

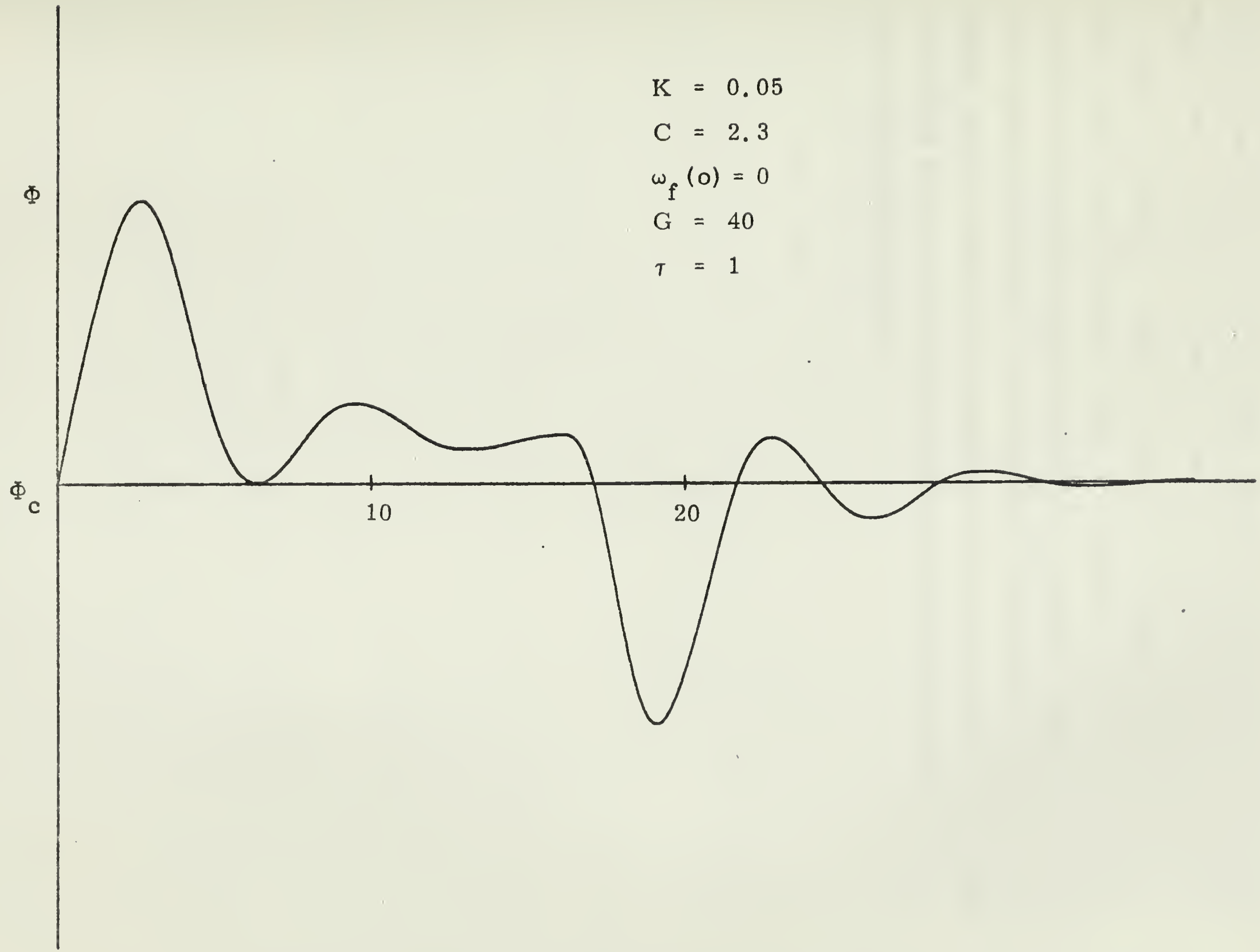


FIGURE (5.19) RESPONSE TO DISTURBANCE TORQUE (SHORT PULSE)

fluid was less than 1% of that of the vehicle. If faster responses were desired the momentum storage capability would have to be increased. As shown in Chapter 3, higher momentum storage requirements result in larger tube diameters and in an increase in the fluid's moment of inertia and system weight. Therefore, for faster responses, the size of the system would have to be increased resulting in increased system weight. Because of this it is important to determine the desired speed of response early in the vehicle design so that the fluid flywheel will not be excessively large.

CHAPTER 6

COMPARISON WITH MECHANICAL FLYWHEEL

6.1 General

As was stated in Chapter 1, the theory of operation of the fluid flywheel is directly applicable to that of the mechanical flywheel. Instead of using a fluid, the angular momentum is stored in a solid and the pump is replaced by a motor. Although no complete study was made of mechanical flywheels, some of the relative merits of their application to attitude control of space vehicles are discussed here.

6.2 Comparison of Weights

The momentum storage capacity of a mechanical flywheel is the product of its moment of inertia and its angular velocity. From this it is obvious that a large capacity can either be achieved by having a large moment of inertia or by having an extremely large angular velocity. This is the same as in the case of the fluid flywheel.

In determining the optimum amount of fluid to be used in the fluid flywheel a trade-off exists between the amount of power to be used and the weight of the fluid. With the fluid flywheel the power losses are extremely high at large angular velocities and therefore the optimum weight was achieved by keeping the angular velocity of the fluid relatively small.

The only losses associated with the mechanical flywheel operating in an evacuated environment are the frictional losses that occur in the bearings. Intuitively it can be seen that these losses will be considerably smaller than those associated with the fluid. Therefore, it

appears that a relatively light flywheel could be used at a very high rpm to achieve the desired angular momentum capacity. Because of this it is felt that the combined weight of a power supply and the flywheel would be less than the combined weight of the same components in the fluid system. It is felt that the weight of the motor and the associated gearing of the mechanical flywheel system would be no more than the weight of the pump and its motor in the fluid system. The weight of the sensors and control circuits would appear to be about the same in both cases. Therefore, it appears that the total system weight of the mechanical flywheel system would be less than the total system weight of the fluid flywheel system for equal momentum storage capabilities.

Reference (1) makes a comparison of the weights of a fluid flywheel and a mechanical flywheel for a particular maximum momentum storage level and finds that the total system weight would be about the same. This report uses brass as the material in the mechanical flywheel and mercury in the fluid flywheel. Brass is a poor choice of material for storing angular momentum. It is felt that if a material with a higher strength density ratio were used in such a comparison that the mechanical flywheel weight would be reduced. Appendix E discusses the merits of using materials with high strength density ratios for momentum storage devices.

6.3 Disadvantages Associated With Mechanical Flywheels

Although mechanical flywheels appear to be lighter than a comparable fluid system, there are many inherent disadvantages associated with them.

The mechanical flywheel requires bearings, a torque motor, and possibly some type of gearing which are all wearing parts. On the other hand, fluid systems have much fewer mechanical moving parts and therefore have greater reliability and life than the mechanical systems. The frictional levels in fluid flywheels, especially when operating at low

momentum storage modes, may be smaller than those associated with the mechanical flywheel. This enables the fluid system to provide both coarse and fine control with the same flywheel. However, most of the systems proposed using the mechanical system require a large flywheel for coarse control and a small one for very precise control around a desired position.

The geometry of the mechanical flywheel requires that it occupy some of the space near the center of the vehicle unless it is located outside the vehicle. Locating it outside the vehicle would introduce additional bearings and other associated mechanical problems, again decreasing its reliability. Before any valid conclusions can be drawn as to whether the mechanical or fluid flywheel is better, a thorough comparative study should be conducted.

CHAPTER 7

CONCLUSIONS

7.1 General

The fluid flywheel system appears to be a feasible means of attitude control for space vehicles. Its principal advantages over other momentum storage devices are its reliability and that its geometry does not require the valuable space located near the center of the vehicle. It is felt that the following trends about fluid flywheel attitude control have been established by this investigation.

1.) The system weight is minimum with fluids of low density and low viscosity.

2.) Low density fluids require larger system volumes.

3.) Good system response can be obtained for a large range of momentum storage levels by using a multiplicative feedback system.

4.) The system can provide attitude control with no steady state position error in the presence of impulses or short pulses in disturbance torques provided that the maximum momentum storage capability of the system is not exceeded.

5.) The minimum system weight is a strong function of the maximum momentum storage capability of the system.

7.2 Other Applications

1.) A small portable fluid flywheel system could be built and attached to people or equipment in the space environment to provide

attitude control on a temporary basis.

2.) The attitude of deep diving vehicles could be controlled by this means. Presently the attitude of submerged vehicles is controlled by fins, trim tanks, and propulsion devices, however the fluid flywheel might have a future application. The tubing could be mounted external to the pressure hull and be fitted with a fluid less dense than water. This could supply buoyancy to the vehicle and at the same time provide the necessary attitude control, while not using the valuable space inside the pressure hull.

7.3 Recommended Areas of Future Study

During the course of this investigation many areas of interest about which little information is known were uncovered. Because the scope of this study was limited these areas were not intensely pursued; however it is felt that efforts in these areas would provide valuable information for future development of fluid flywheel attitude control systems.

1.) Disturbance torques -

Although a great deal of work has been done investigating the size and frequency of occurence of disturbance torques, only rough estimates of the momentum storage capacity for the fluid flywheel are possible. It is felt that more effort should be devoted to the determination of the magnitude of the cyclic and non-cyclic disturbance torques. The size of these torques affects the momentum storage capacity of the flywheel and therefore the total system weight, as well as the size of the auxiliary mass expulsion system.

2.) Dumping stored momentum -

The fluid losses are large when the system is operating in a high momentum storage mode, requiring large power consumption. It is desirable to develop an effective means of dumping stored momentum

without using the auxiliary mass expulsion system. This would result in a smaller power supply and reduce the size of, or possibly eliminate, the auxiliary mass expulsion system. This could possibly be done by placing the vehicle in an attitude in which a predicted disturbance torque would be counteracted by a deceleration of the fluid in the flywheel.

3.) Nondimensionalized information -

It would be of assistance to the designer if the type of information in this report could be nondimensionalized and presented in graphical form. The speed of response, natural frequencies, and other system parameters may be able to be plotted as functions of the momentum storage requirements and the weight of the flywheel relative to the weight and moment of inertia of the vehicle.

4.) Pumping devices -

It is felt that the future applications of this system are greatly dependent on the development of an effective way to pump the fluid. A thorough investigation of the techniques mentioned in Chapter 4 and other feasible means of pumping fluids should be conducted, keeping in mind the necessity of few moving parts and no leakage.

5.) Comparative Study -

A thorough study comparing the fluid and mechanical attitude control systems should be conducted. The study should investigate the recent developments of high performance flywheels constructed of glass-reinforced plastic or beryllium and which use gas bearings instead of the more conventional types. If these flywheels could be used in the control system the size of the power supply could be reduced. These flywheels are presently being developed to replace conventional catapults on aircraft carriers.

6.) Magnetic fluids -

Quite recently a great deal of work has been done in-

vestigating fluids which exhibit magnetic properties. These fluids can be attracted by magnetic fields. Possibly these fluids could be used in a fluid flywheel application being pumped by magnetic fields.

7.) Combined power supply and control system -

There are several power supplies being developed for use in space which are classified as Solar Dynamic and Nuclear Dynamic systems. These devices use either solar or nuclear radiation to heat a two phase fluid which drives some type of vapor turbine coupled to an electrical generator, thus producing power. It may be possible to use this fluid in the fluid flywheel to produce the control torques and at the same time the flywheel could serve as the condensor for the vapor cycle. Some of these systems have specific weights of less than 0.1 lbs/watt making them an attractive power supply. This coupled with the fact that the fluid would serve a dual purpose would make the combined power supply and attitude control system weight relatively low.

8.) Optimization of dynamic performance -

A study similar to the one in Chapter 5 should be conducted considering all three axes at the same time. The interaxis coupling effects produced by movements about one axis will act as disturbance torques about the other axes. The magnitude of these torques is dependent upon the vehicle geometry and mass distribution, loop configuration, and the momentum storage levels in the loops. The entire system should be optimized to obtain good responses considering these dynamic interactions of the loops as well as the dynamic affects of momentum dumping.

APPENDIX

APPENDIX A

LOSSES IN LOOP

The losses in the loop consist of the following: the frictional losses due to the interaction of the fluid and the tubing, losses due to any restrictions in the loop, and losses directly associated with the pumping device. Assuming that the loop is of circular configuration, that there are no restrictions in the tube, and that the losses associated with the pump are included in the pump overall efficiency, the losses can be calculated as a function of fluid velocity.

The pressure drop for a length of pipe using the Darcy-Weisbach form is given by the following:

$$\Delta p = \frac{f\,l}{D}\,\frac{V^2}{2g}\,\gamma_f \qquad \text{(A 1)}$$

Substituting l for a circular configuration of diameter D_1:

$$\Delta p = \frac{f\,\gamma_f\,\pi\,D_1\,V^2}{2\,D_2\,g} \qquad \text{(A 2)}$$

The values for Δp for different fluids for the range of velocity from 0 to 25 feet per second are shown in Figure (A. 1). These values were obtained using an estimate of the tube diameter to be used in the optimization study. The diameters were estimated by keeping the moment of inertia of the fluid constant as the density of the fluid was changed. The values used here are actually quite close to the values obtained in the optimization study for $H_{MAX} = 10000$, and $D_1 = 10$. The friction factors were obtained using figures for a smooth pipe from Reference (8).

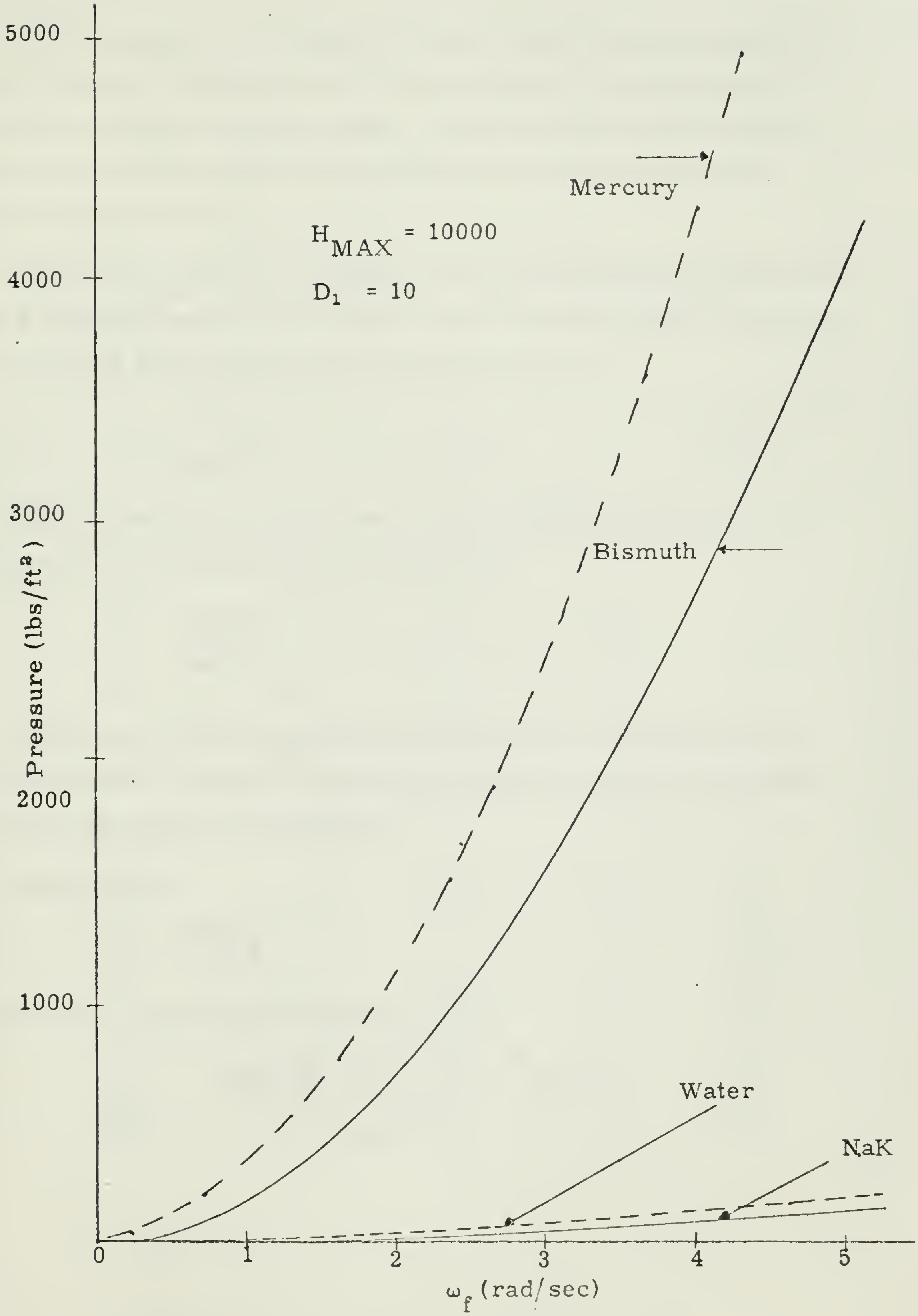


FIGURE (A.1) PRESSURE DROP IN LOOP

For the purposes of the optimization study it was desirable to develop an analytic function for the friction factor. It was found that for all of the fluids, through the range, that the friction factor times the Reynold's number raised to the 0.16 power was a constant. See tables (A-1 through A-4).

To allow for a factor of safety to make sure that the losses were not being underestimated this constant was increased slightly and in the rest of the study the friction factor was assumed to be:

$$f = \frac{.11}{Re^{.16}} \tag{A 3}$$

This corresponds quite closely to the approximation given in Reference (9) if the constant is divided by four.

$$f = \frac{.046}{Re^{.2}}$$

The factor of four is introduced because the fraction factor is defined differently, (Moody), and changes nothing as long as the factor is used with the appropriate definition.

Substituting

$$V = \frac{D_1 \omega_f}{2}$$

and the friction factor approximation in (A 2):

$$\Delta p = \frac{.0483\, \rho_f^{.84}\, D_1^{2.84}\, \mu^{.16}\, \omega_f^{1.84}}{D_2^{1.16}} \tag{A 4}$$

TABLE A-1

PRESSURE DROP IN LOOP

MERCURY TUBE DIA .345 ft. (SMOOTH PIPE)

V ft/sec	Re	f	Δp #/ft²	$Re^{.16}$	$f\, Re^{.16}$
1	2.81×10^5	.0145	17.4	7.45	.108
2	5.62	.0129	62.0	8.46	.109
3	8.43	.012	129.5	8.9	.107
4	11.24	.0113	207.0	9.3	.105
5	14.05	.0110	313.0	9.65	.106
10	28.1	.0096	1150.0	10.8	.104
15	42.1	.0091	2460.0	11.5	.105
20	56.2	.0088	4230	12.1	.107
25	70.25	.0085	6360	12.5	.1065

TABLE A-2

PRESSURE DROP IN LOOP

BISMUTH DIA = .407 ft. (USING SMOOTH PIPE)

V	Re	f	p $^{\#}/_{ft^2}$	$Re^{.16}$	$f\ Re^{.16}$
1	2.3×10^5	.015	11.02	7.22	.108
2	4.6	.0131	33.2	8.06	.105
3	6.9	.0124	82.0	8.61	.107
4	9.2	.0118	131.0	9.0	.106
5	11.5	.0113	197.0	9.33	.105
10	23.0	.0101	743.0	10.5	.106
15	34.5	.0095	1570	11.1	.106
20	46.0	.009	2643	11.6	.105
25	57.5	.0087	4000	12.1	.105

TABLE A-3

PRESSURE DROP IN LOOP

NaK (23% Na - 77% K) DIA = 1.351 ft. (SMOOTH PIPE)

V $^{ft}/_{sec}$	Re	f	P_{loss} $^{\#}/_{ft^2}$	$Re^{.16}$	f $Re^{.16}$
1	1.89×10^5	.0153	.306	7.0	.107
2	3.78	.0136	1.09	7.8	.106
3	5.67	.0129	2.32	8.35	.108
4	7.56	.012	3.84	8.75	.105
5	9.45	.0118	5.9	9.07	.107
10	18.9	.0104	20.8	10.1	.105
15	28.3	.0096	43.2	10.8	.104
20	37.8	.0094	75.4	11.25	.106
25	47.25	.009	112.7	11.7	.105

TABLE A-4

PRESSURE DROP IN LOOP

WATER DIA = 1.275 ft. (USING SMOOTH PIPE)

V ft/sec	Re	f	P_{loss} #/ft²	$Re^{.16}$	f $Re^{.16}$
1	1.035×10^5	.0176	.42		
2	2.07	.0151	1.505	7.1	.107
3	3.10	.0141	3.16	7.6	.107
4	4.14	.0136	5.43	7.95	.108
5	5.17	.0130	8.12	8.3	.108
10	10.35	.0115	27.4	9.2	.106
15	15.5	.0107	60.0	9.84	.105
20	20.7	.0101	101.0	10.3	.104
25	25.8	.0098	155.0	10.65	.1045

APPENDIX B

COMPONENT WEIGHTS

Fluid Weight

$$W = (\gamma_f)\ (\text{Volume})$$

$$W = \gamma_f \frac{\pi^2 D_1 D_2^2}{4} \qquad (B\ 1)$$

Tubing Weight

Tubing thickness = t a constant (See Appendix C)
assumed .01 inches

$$W = \pi^2 D_1 D_2 t \gamma_m \qquad (B\ 2)$$

Weight of Power Supply

assuming: average power = .2 Power_{MAX}

specific weight = .225 lb/watt

$$P = \Delta p Q \qquad (B\ 3)$$

$$P = \frac{f \gamma_f \pi D_1 V^2}{2 D_2 g} Q \qquad (B\ 4)$$

where $Q = \dfrac{V \pi D_2^2}{4}$

$$P = \frac{f \pi^2 \rho_f D_1 D_2 V^3}{8} \qquad (B\ 5)$$

$$V = \frac{D_1 \omega}{2} \qquad (B\ 6)$$

$$P = \frac{f \pi^2 \rho_f D_1^4 D_2 \omega^3}{64} \qquad (B\ 7)$$

From equation (A 3)

$$f = \frac{.11}{Re^{.16}},$$

where

$$Re = \left(\frac{D_2 V}{\nu}\right) = \left(\frac{D_2 D_1 \omega}{2\nu}\right)$$

Substituting (B 7) for F:

$$P = \frac{(2)^{.16}(.11)\pi^2}{64} \rho_f D_1^{3.84} D_2^{.84} \nu^{.16} \omega^{2.84} \qquad \text{(B 8)}$$

Maximum power will occur with maximum ω_f, for specific value of H_{MAX}:

$$\omega_{MAX} = \frac{H_{MAX}}{I_f} \qquad \text{(B 9)}$$

Assuming circular configuration for the fluid loop:

$$I_f = \frac{\pi^2 D_2^2 D_1^3 \rho_f}{16} \qquad \text{(B 10)}$$

Substituting (B 10) in (B 9)

$$\omega_{MAX} = \frac{16 H_{MAX}}{\pi^2 D_2^2 D_1^3 \rho_f} \qquad \text{(B 11)}$$

Substituting (B 11) in (B 8)

$$P_{MAX} = \frac{.0737 \nu^{.16} H_{MAX}^{2.84}}{\rho_f^{1.84} D_2^{4.84} D_1^{4.68}} \qquad \text{(B 12)}$$

$$W = \left(\frac{.225}{.737}\right)(.2) P_{MAX} =$$

$$= \frac{(.045) \nu^{.16} H_{MAX}^{2.84}}{\rho_f^{1.84} D_1^{4.68} D_2^{4.84}} \qquad \text{(B 13)}$$

Weight of pump motor

$$\text{Weight} = .468 (P_{MAX})^{.7} \qquad \text{(B 14)}$$

Fitting the curve from Reference (1), and assuming pump is 50% efficient.

Weight of Pumping Device

$$W = 500 D_2^3 \tag{B 15}$$

APPENDIX C

DETERMINATION OF TUBING THICKNESS

Assuming that the tubing is not going to be a structural member of the vehicle the stresses caused by the fluid will determine the tubing thickness. The maximum stresses will occur during lift off or during the maximum momentum storage mode. Here it is assumed that the vehicle control system is not in use during lift off, so that the larger of these controls the tubing thickness.

During Lift Off

For large loop diameters and small tube diameters the tubing can be approximated as a circular beam. Under normal circumstances the loading is just the weight per foot of tubing. However during lift off, due to the acceleration of the vehicle, the loading increases. The loading is equal to the mass times the acceleration. (See Figure C. 1)

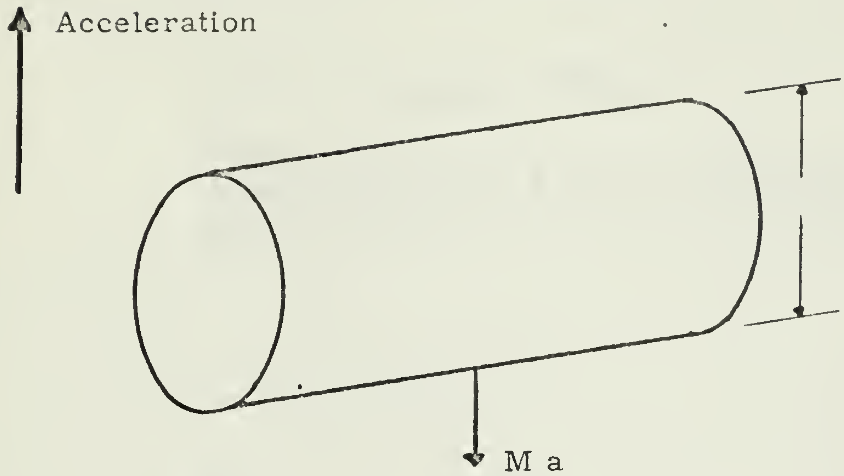
$$\frac{\text{Force}}{\text{Unit length}} = \frac{M\ a}{\text{Unit length}}$$

$$w = \frac{\pi D_2^2 \rho a}{4} \qquad \text{(C 1)}$$

Assuming that the tubing is supported by clamped supports a distance l apart and that D_1 is large relative to D_2 so that the beam can be considered straight between the supports, the bending moment can be calculated as follows. The maximum bending moment occurs at the clamped supports. (See Figure C. 2)

$$M_b = \frac{\pi D_2^2 \rho a l^2}{48} \qquad \text{(C 2)}$$

FORCES ON ELEMENTAL MASS



TUBING CROSS SECTION

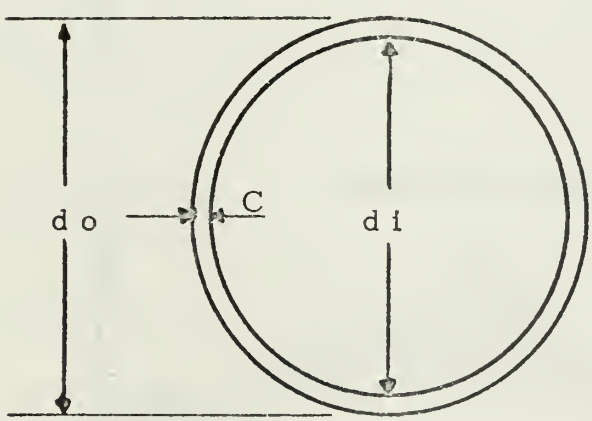


FIGURE (C. 1)

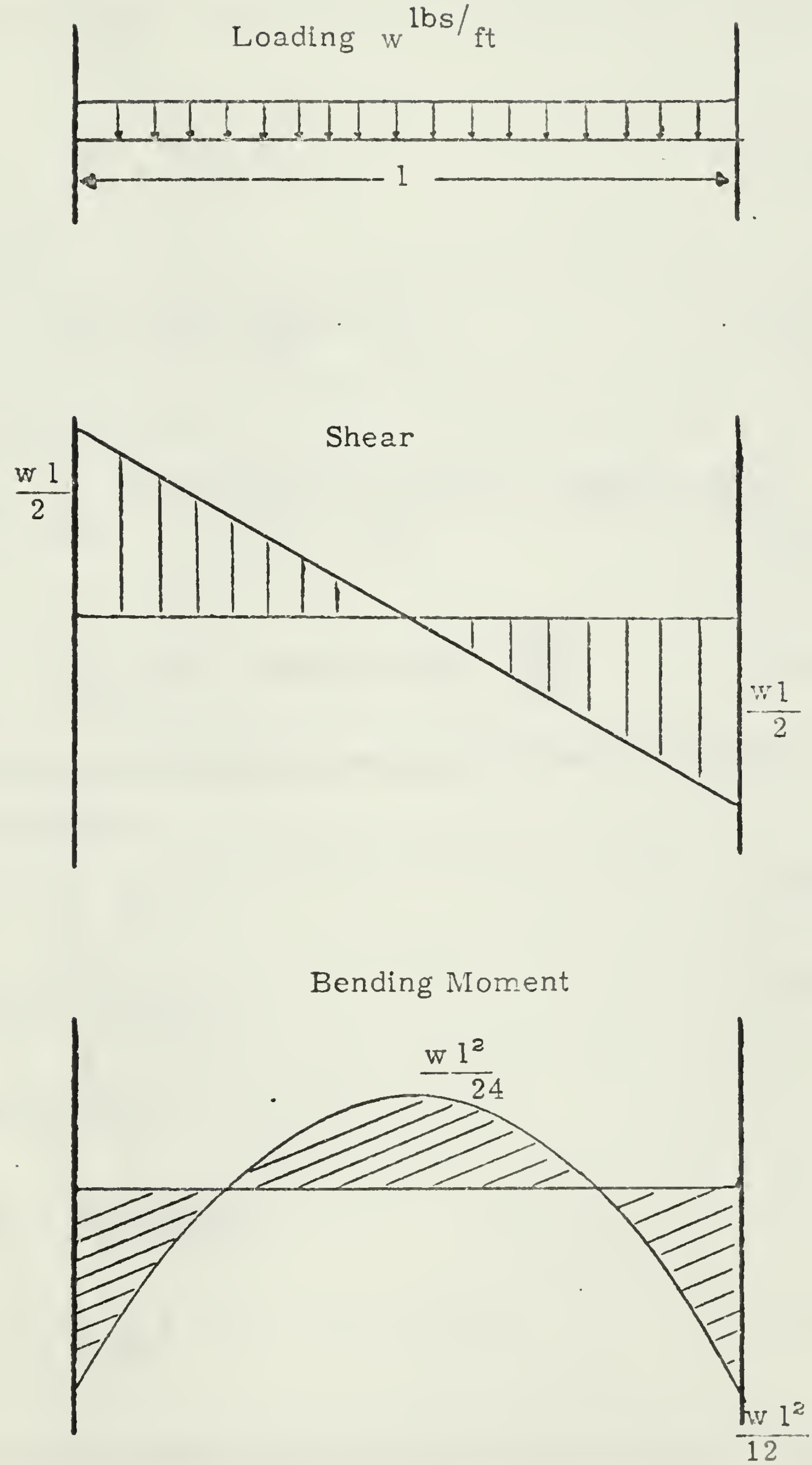


FIGURE (C. 2) LOADING ANALYSIS

The maximum stresses from this are in the fibers on the upper and lower surfaces of the tubing.

$$S_t = \frac{M_b}{Z} \qquad (C\ 3)$$

where

$$Z = \frac{\pi}{32} \left(\frac{d_o^4 - d_i^4}{d_o} \right) \qquad (C\ 4)$$

substituting for t,

$$Z = \frac{\pi}{32} \left(\frac{D_2^4 - (D_2 - 2\,t)^4}{D_2} \right) \qquad (C\ 5)$$

Expanding the second term,

$$Z = \frac{\pi}{32} \left(\frac{D_2^4 - (D_2^4 - 8D_2^3 t + 24D_2^2 t^2 - 32D_2\, t^3 + 16t^4)}{D_2} \right) \qquad (C\ 6)$$

$$Z = \frac{\pi}{32} \left(8D_2^2 t - 24D_2 t^2 + 32t^3 - \frac{16t^4}{D_2} \right) \qquad (C\ 7)$$

Since t is small, all the terms involving t raised to a power higher than one are neglected.

$$Z \approx \frac{\pi}{4} D_2^2\, t \qquad (C\ 8)$$

$$S_t = \frac{M_b}{\frac{\pi}{4} D_2^2\, t} \qquad (C\ 9)$$

Substituting for M_b,

$$S_t = \frac{\pi D_2^2 \rho\, a\, l^2}{12\, \pi\, D_2^2\, t}$$

$$S_t = \frac{\rho\, a\, l}{12\, t} \qquad (C\ 10)$$

Using this approximation the stress caused by bending is constant for any diameter.

Shear Stresses in Tubing

The maximum shear stresses in the tubing will occur at the supports and will occur in the fibers half-way down the sides of the tubing.

$$\tau = \frac{V\,Q}{I\,b} \qquad \text{(C 11)}$$

where b is the thickness

Q is the first moment of area above the section examined with respect to the neutral axis

I is the moment of inertia of the entire section

V is the shear force acting on that section.

From Figure (C.2) V is equal to wl/2

$$V = \frac{\pi D_2^2 \rho a l}{8}$$

$$Q = 2t\,D_2$$

$$I = \frac{\pi}{32}(D_0^4 - D_1^4)$$

Using the same expansion technique as used in (C 6),

$$I \approx \frac{\pi}{4} D_2^3\, t \qquad \text{(C 12)}$$

Substituting into (C 11),

$$\tau = \frac{\pi D_2^2 \rho\; a\; l\,(t\,D_2)}{4\frac{\pi}{4} D_2^3\, t\,(2t)}$$

$$\tau = \frac{\rho\; a\; l}{2\,t} \qquad \text{(C 13)}$$

Again this is independent of the tube diameter.

The maximum normal stress may occur some place in the tubing between the location of the maximum tensil stress and the maximum shear stress. This also, with the approximations made above, will be

independent of the tube diameter.

Maximum Stresses During Momentum Storage Mode of Operation

When the fluid is being pumped at its maximum velocity it produces two types of loading on the tubing. First, since the pump output is at the maximum pressure the hoop stresses will be maximum. Second, since the fluid velocity is maximum it produces the maximum centrifugal loading causing bending moments on the tubing.

The bending moments are handled in exactly the same manner that they were handled in the lift off case except that the uniform loading is $\frac{MV^2}{r}$ and now the tensil stresses are rotated 90 degrees.

$$w = \frac{MV^2}{r} = \frac{\rho \, \pi \, D_2^2 \, V^2}{2 \, D_1} \tag{C 14}$$

$$V = \frac{\omega_f \, D_1}{2}$$

$$\omega_{MAX} = \frac{H_{MAX}}{I_f}$$

$$V = \frac{H_{MAX}}{I_f} \; \frac{D_1}{2}$$

Substituting into (C 14),

$$w = \frac{\rho \, \pi \, D_2^2 \, H_{MAX}^2 \, D_1}{8 \, I_f^2} \tag{C 15}$$

$$I_f = Mr^2 = \frac{\pi^2 D_2^2 \, D_1^3 \, \rho}{16} \tag{C 16}$$

$$w = \frac{H_{MAX}^2 \, (32)}{\pi^3 D_2^2 \, D_1^5 \, \rho}$$

$$M_b = \frac{8 \, H_{MAX}^2 \, 1^2}{3 \, \pi^2 \, D_2^2 \, D_1^5 \, \rho} \tag{C 18}$$

$$S_t = \frac{32}{3} \frac{H_{MAX}^2 \; l^3}{\pi^4 \; D_2^4 \; D_1^5 \; \rho \, t} \tag{C 19}$$

Maximum Shearing Stress

$$\tau = \frac{V\,Q}{I_b} = \frac{64\, H_{MAX}^2 \; l}{\pi^4 \; D_1^5 \; \rho \, t \, D_2^4} \tag{C 20}$$

Maximum Hoop Stresses

$$S = \frac{F}{A} = \frac{\Delta p \, D_2}{2\, t} \tag{C 21}$$

$$S_t = \frac{f\, 64\, H_{MAX}^2}{\pi^3 \, \rho \, D_2^4 \; D_1^3 \quad t} \tag{C 22}$$

The maximum normal stress, the resultants of (C 14), (C 20), and (C 22) has to be determined using a Mohrs Circle, however it is obvious that this will decrease rapidly with increasing tube diameter.

Summary

It is not obvious here whether the stress during life-off or the stress during maximum momentum storage will govern the tubing thickness. This would depend on the actual amount of momentum to be stored and on the acceleration of the vehicle during lift off. However it can be seen that if the thickness for the minimum tube diameter is satisfactory, it will be satisfactory as the tube diameter is increased.

These stresses were calculated for the case of tubing filled with mercury at its optimum tube diameter and supported every foot. It was found that for H_{MAX} = 10,000 ft-lbs-sec^2, if stainless steel were used for the tubing, the thickness was only a few thousandths of an inch. Because of this the actual thickness of the tubing might be imposed by manufacturing limitations. For the purpose of this study it was assumed

to be constant. Materials with high strength density ratios are the best materials to use for the tubing for the purpose of reducing system weight, making stainless steel and fiberglass excellent for this purpose.

APPENDIX D

DERIVATION OF SYSTEM FUNCTIONS

Referring to Figure (5.2) and Figure (5.3) the frictional feedback loop can be reduced to:

$$\frac{1}{1 + \dfrac{K}{I_f s}} \tag{D 1}$$

This simplifies to:

$$\frac{I_f s}{K + I_f s} \tag{D 2}$$

Incorporating this into the velocity feedback loop it becomes:

$$\frac{\left(\dfrac{s + \frac{1}{\tau}}{s}\right)\left(\dfrac{G_p I_f s}{K + I_f s}\right)\dfrac{1}{I_v s}}{1 + C\left(\dfrac{s + \frac{1}{\tau}}{s}\right)\left(\dfrac{G_p I_f s}{K + I_f s}\right)\dfrac{1}{I_v s}} \tag{D 3}$$

Simplifying:

$$\frac{(s + \frac{1}{\tau})\ G_p I_f}{I_f I_v s^2 + (K I_v + C I_f G_p)s + \frac{C}{\tau} G_p I_f} \tag{D 4}$$

Adding in the position feedback loop:

$$\frac{\dfrac{(s + \frac{1}{\tau})\ G_p I_f}{I_f I_v s^2 + (K I_v + C I_f G_p)s + \frac{C}{\tau} G_p I_f}}{1 + \dfrac{(s + \frac{1}{\tau})\ (G_p I_f)}{I_f I_v s^2 + (K I_v + C I_f G_p)s + \frac{C}{\tau} G_p I_f}} \tag{D 5}$$

Simplifying:

$$\frac{\Phi}{\Phi_c} = \frac{(s + \frac{1}{\tau})\ G_p I_f}{I_v I_f s^3 + (KI_v + CI_f G_p)s^2 + (\frac{C}{\tau} G_p I_f + G_p I_f)s + \frac{G_p I_f}{\tau}}$$

$$G_p = G \qquad \text{(D 6)}$$

<u>Steady State Error</u>

In a similar manner the error function for inputs for T_D is derived to be:

$$e = \frac{-T_D (I_f C s^2 + (CK + I_f)s + K)}{I_v I_f s^3 + (KI_v + GI_f C)s^2 + (GI_f + \frac{CGI_f}{\tau})s + \frac{GI_f}{\tau}}$$

For an impulse at T_D and using the final value theorem:

$$\lim_{s \to 0} s\,F(s) = F(t) \text{ at infinity}$$

$$e(\infty) = \lim_{s \to 0} \frac{-sT_D (I_f C s^2 + (CK + I_f)s + K}{I_v I_f s^3 + (KI_v + GI_f C)s^2 + (GI_f + \frac{CGI_f}{\tau})s + \frac{GI_f}{\tau}} \qquad \text{(D 7)}$$

$$e(\infty) = \frac{0}{\frac{G I_f}{\tau}} = 0$$

For a step at T_D:

$$e(\infty) = \frac{K}{GI_f/\tau}$$

<u>Error for Step Input at Φ_c</u>

It is obvious that for inputs at Φ_c the steady state error is zero because of the free integration in the forward loop. See Reference (6).

APPENDIX E

MATERIALS FOR MECHANICAL FLYWHEEL

The limit on the momentum storage capacity for a mechanical flywheel of any configuration is imposed by the strength of the materials used in its fabrication. This occurs because the limiting speed is determined by the working stress of the material. The stresses may be analyzed by considering a thin loop as shown in Figure (E. 1)

For equilibrium:

$$2 (T \sin \frac{\varphi}{2} = \omega^2 \; r \; dm \qquad (E\ 1)$$

If the loop is large and $d\varphi$ is small, then:

$$\sin \frac{d\varphi}{2} = \frac{d\varphi}{2} \qquad (E\ 2)$$

$$dm = \rho \; A \; r \; d\varphi \qquad (E\ 3)$$

Therefore:

$$2 \; T \; \frac{d\varphi}{2} = \omega^2 r^2 \; \rho \; A \; d\varphi$$

$$\omega^2 \; r^2 = \frac{T}{\rho A} \qquad (E\ 4)$$

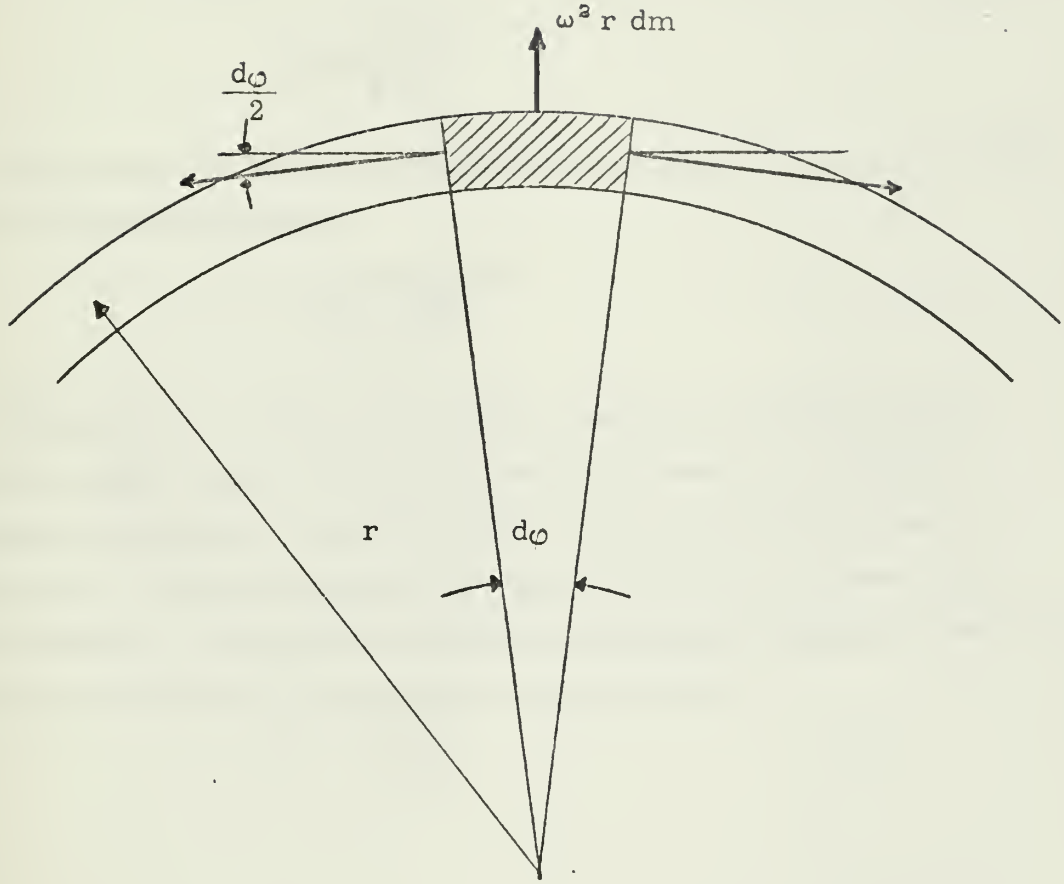
$\frac{T}{A}$ is the stress in the material.

If the maximum working stress is taken to be S_0 then:

$$\omega^2 \; r^2 = \frac{S_0}{\rho} \qquad (E\ 5)$$

$$\omega_{MAX} = \frac{1}{r} \sqrt{\frac{S_0}{\rho}}$$

$$I = m \; r^2 \qquad (E\ 6)$$



T = tensile force
A = Cross sectional area
ρ = Mass density of material

FIGURE (E. 1) FORCES ACTING ON MECHANICAL FLYWHEEL

Therefore:

$$
\begin{aligned}
H_{MAX} &= I\,\omega_{MAX} \\
&= mr\sqrt{\frac{s_o}{\rho}}
\end{aligned}
\qquad (E\ 7)
$$

Reference (11) derives an expression similar to Equation (E 5) for disks of uniform stress.

$$
\omega = \frac{1.3}{R}\sqrt{\frac{2\,s_o}{\rho}}
$$

From this it is obvious that with materials with high strength density ratios make the best mechanical flywheels for storage of angular momentum. Materials such as glass-reinforced plastic or beryllium, although they are less dense than brass or steel, have much higher working stress limits and therefore flywheels made from these materials are more suited for this purpose.

APPENDIX F

LIST OF SYMBOLS

H	Angular momentum	ft-lbs/sec
I_v	Moment of inertia of vehicle	ft-lbs-sec^2
I_f	Moment of inertia of fluid	ft-lbs-sec^2
ω_v	Angular velocity of vehicle	rad/sec
ω_f	Angular velocity of fluid	rad/sec
Φ_v	Angular position of vehicle	rad
Φ_f	Angular position of fluid	rad
D_1	Fluid flywheel loop diameter	ft
D_2	Tubing diameter	ft
γ_f	Specific weight of fluid	lbs/ft^3
γ_m	Specific weight of metal	lbs/ft^3
t	Tubing thickness	ft
ρ_f	Density of fluid	lbs-sec^2 /ft^4
μ	Dynamic viscosity	lbs-sec/ft^2
ν	Kinematic viscosity	ft^2 /sec
V_f	Fluid velocity	ft/sec
s	Laplace Operator	1/sec
T_P	Pump torque	ft-lbs
T_L	Loop losses torques	ft-lbs
T_N	Net torque	ft-lbs
Δp	Pressure drop in loop	lbs/ft^3
K	Linearized feedback constant	ft-lbs-sec
G	Pump gain	ft-lbs/rad
f	Darcy-Weisbach friction factor	
Re	Reynolds number	

W	Weight	lbs
P	Power	ft-lbs/sec
a	acceleration	ft/sec^2
α	angular acceleration	rad/sec^2
s_o	Maximum working stress	lbs/ft^2

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